IV - 1. An analog signal $x_a(t) = \sin(2\pi f t)$ with $f = 100$ Hz is sampled with period:

a) $T = 2.5$ mS

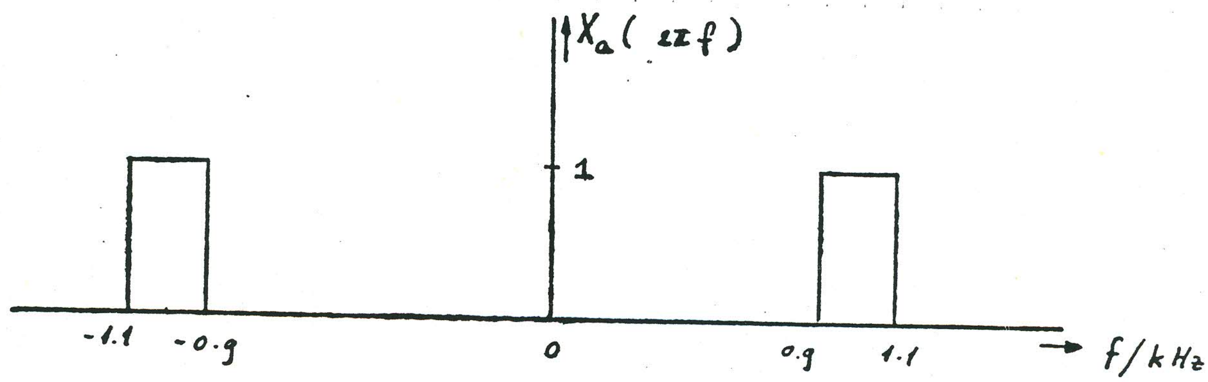
b) $T = 5.0$ mS

c) $T = 10$ mS.

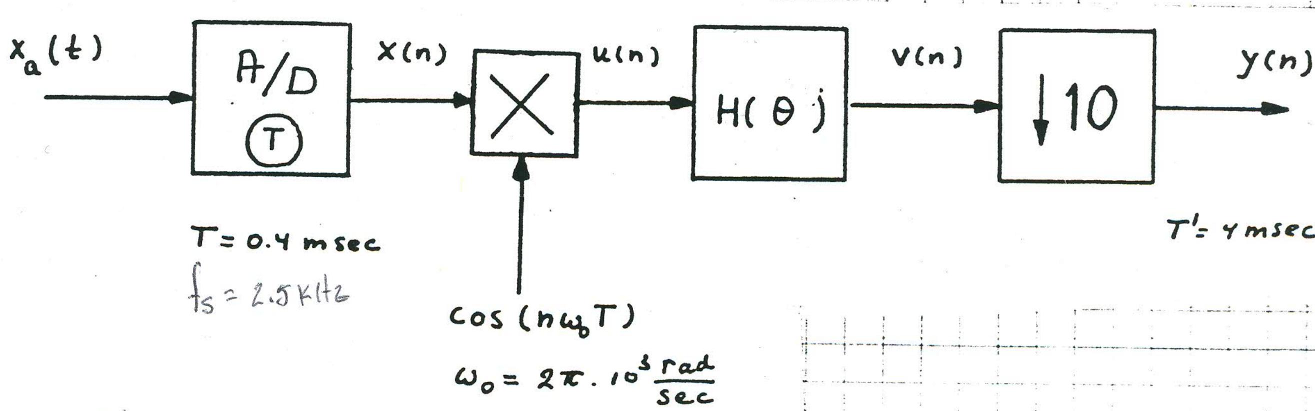
For each of these cases determine the output signal of the A/D convertor.

Repeat the computation for $x_a(t) = \cos(2\pi f t)$.

IV - 2. The analog signal $x_a(t)$ has the spectrum $X_a(\omega)$ shown below.



It is used as input signal for the following system:



In this system $H(\theta)$ is an ideal low-pass filter with cut-off frequency $f_c = 125$ Hz.

a) Determine the spectra of the signals $x(n)$, $u(n)$, $v(n)$, $y(n)$.

b) Show that it is possible to obtain $y(n)$ by sampling $x_a(t)$ with period $T' = 4$ msec.

IV – 3. A digital signal $x(n)$ having a sampling period T has to be converted into an analog signal $x_a(t)$ in such a way that:

$$X_a(\omega) = \begin{cases} X(\omega T) & |\omega| < \pi/T \\ 0 & |\omega| \geq \pi/T \end{cases}$$

We have available a non ideal D/A convertor that delivers the output signal

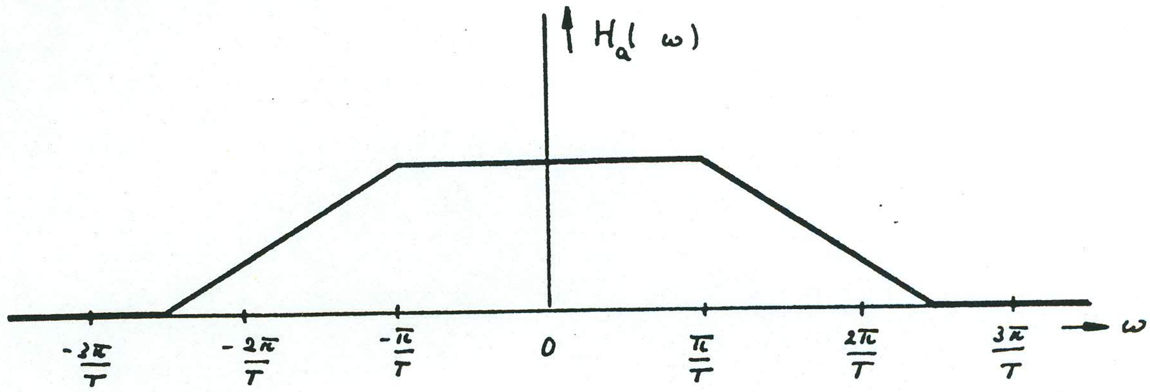
$$y_a(t) = \sum_{n=-\infty}^{\infty} x(n)\, p(t-nT) = x(t) * p(t)$$

where

$$p(t) = \begin{cases} 1 & |t| < T/4 \\ 0 & |t| \geq T/4 \end{cases}$$

when $x(n)$ is applied to it.

Moreover, we have an analog filter H_a with transmission function shown below to do the filtering after D/A conversion.



What kind of digital pre-processing is required to obtain the desired output spectrum?

IV - 4. For the digital signal $x(n) = a^n u(n)$, $|a| < 1$

a) determine the spectrum.

b) $x(n)$ is applied to a sampling rate decrease by a factor 2. Determine the output spectrum using the frequency domain description of the SRD.

c). Show that this spectrum is the spectrum of $x(2n)$

IV - 5. $x(n)$ is a digital signal that has the sampling period T. From this signal a new signal is derived having sampling period $T/2$ by a linear interpolation:

$$y(n) = \begin{cases} x(n/2) & n \text{ even} \\ \frac{1}{2}\left[x\left(\frac{n-1}{2}\right) + x\left(\frac{n+1}{2}\right)\right] & n \text{ odd} \end{cases}$$

a) Show that this linear interpolation can be realized by digital signal processing elements

b) Determine the spectrum of $y(n)$ for the case that the spectrum of $x(n)$ equals

$$X(\omega T) = \begin{cases} 1 & \frac{\omega_0 - \Delta\omega}{T} < |\omega| < \frac{\omega_0 + \Delta\omega}{T} \\ 0 & \text{elsewhere} \end{cases}$$

where

$$\omega_0 = \Delta\omega = \frac{\pi}{10T}$$

c) Repeat the computation for

$$\omega_0 = \frac{4\pi}{5T}, \quad \Delta\omega = \frac{\pi}{10T}$$