II - 1. A causal system is described by the difference equation:

$$y(n) - \frac{1}{2}\, y(n-1) = x(n) + \frac{1}{2}\, x(n-1),$$

where $y(n)$ is the output and $x(n)$ the input of the system.

a Show an implementation of this system.

b Determine the impulse response $h(n)$ of the system.

c Determine the system response to the excitation:

$$x(n) = e^{j\theta n}$$

using the result of b.

d Determine the transmission function.

e Determine the response to

$$x(n) = \cos\left(n \cdot \frac{\pi}{2} + \frac{\pi}{4}\right).$$

II - 2. Let

$$x(n) = \begin{cases} 0 & |n| > 2 \\ 1 & n = \pm 2 \\ 2 & n = \pm 1 \\ 3 & n = 0 \end{cases}$$

a Determine the Fourier transform of $x(n)$.

b Write $x(n)$ as the convolution of two signals $x_1(n)$ and $x_2(n)$.

c Again determine the FTD of $x(n)$, but now using the FTD's of $x_1(n)$ and $x_2(n)$.

II-3 For an ideal band pass filter the frequency response in the fundamental interval $(-\pi, \pi)$ is given by:

$$H(\theta) = \begin{cases} 0 & |\theta| \leq \frac{\pi}{8} \\ 1 & \frac{\pi}{8} < |\theta| < \frac{3\pi}{8} \\ 0 & \frac{3\pi}{8} \leq |\theta| \leq \pi \end{cases}$$

a Determine the corresponding impulse response

b Show that this impulse response can be written as the product of $\cos n\frac{\pi}{4}$ and the impulse response of a low pass filter.