

## Digital Signal Processing

### Fast Fourier Transform

I-1. a. Determine the convolution of the two discrete signals:

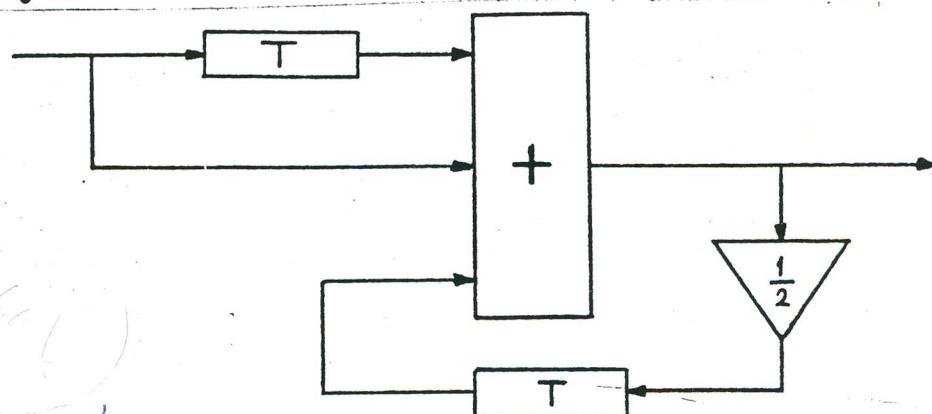
$$x(n) = \begin{cases} 0 & n < 0 \\ 1 & 0 \leq n \leq 3 \\ 0 & n > 3 \end{cases}$$

$$h(n) = \begin{cases} 0 & n < 0 \\ 1 & 0 \leq n \leq 4 \\ 0 & n > 4 \end{cases}$$

b. As a, except with

$$h(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

I-2. a. Determine the impulse response  $h(n)$  of the system below:



b Show that  $h(n)$  can be taken as the convolution of  $h_1(n)$  and  $h_2(n)$  where

$$h_1(n) = \begin{cases} 0 & n < 0 \\ 1 & 0 \leq n \leq 1 \\ 0 & n > 1 \end{cases}$$

$$h_2(n) = \begin{cases} 0 & n < 0 \\ (\frac{1}{2})^n & n \geq 0 \end{cases}$$

I-3 The system  $h$  consists of a cascade of two linear time invariant systems  $h_1$  and  $h_2$

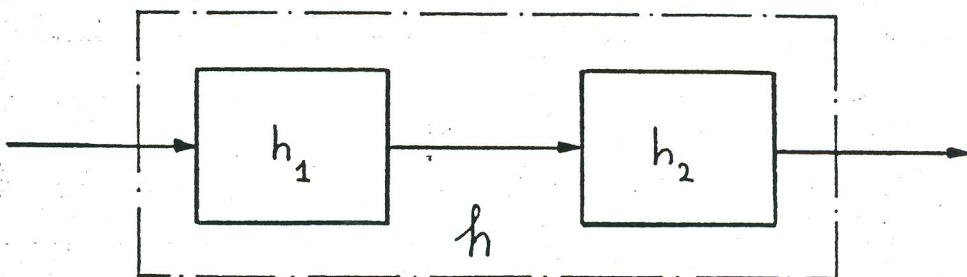
a Proof that  $h$  is stable when  $h_1$  and  $h_2$  both are stable

b Proof that  $h$  is causal when  $h_1$  and  $h_2$  both are causal

c Show with an example that the inverse does not hold in general, i.e.

$h$  stable  $\rightarrow h_1$  and  $h_2$  stable

$h$  causal  $\rightarrow h_1$  and  $h_2$  causal.



I-4. a Determine for each of the three systems below the impulse response.

b Will it be possible to choose the coefficients  $(a_0, a_1, a_2)$ ,  $(b_0, b_1, b_2)$  and  $(c_1, c_2, c_3)$  in such a way that the impulse responses will be identical?

