2.1

Analog to digital/Digital to analog

Digital signals:

- Originate from analog signals
- Are generated by digital generators

Theoretical description of an A/D converter

$$x_a(t) \rightarrow \boxed{\text{A/D}} \rightarrow x(n)$$

$$X_a(\omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\omega t} dt \qquad X(\theta) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\theta}$$

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(\omega) e^{j\omega t} d\omega \qquad x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\theta) e^{jn\theta} d\theta$$

Time-domain description of the ideal A/D convertor:

$$x(n) = x_a(nT)$$

Frequency domain description? ⇢ analog

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2.2

$$x_a(nT) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(\omega) e^{j\omega nT} d\omega \qquad \omega T = \theta$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a\left(\frac{\theta}{T}\right) e^{jn\theta} \frac{d\theta}{T} = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \frac{1}{T} \int_{k2\pi-\pi}^{k2\pi+\pi} X_a\left(\frac{\theta}{T}\right) e^{jn\theta} d\theta \qquad \theta \rightarrow \theta + 2k\pi$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \frac{1}{T} \int_{-\pi}^{\pi} X_a\left(\frac{\theta + 2k\pi}{T}\right) e^{jn(\theta + 2k\pi)} d(\theta + 2k\pi)$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \frac{1}{T} \int_{-\pi}^{\pi} X_a\left(\frac{\theta + 2k\pi}{T}\right) e^{jn\theta} d\theta$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{\frac{1}{T} \sum_{k=-\infty}^{\infty} X_a\left(\frac{\theta + 2k\pi}{T}\right)\right\} e^{jn\theta} d\theta$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\theta) e^{jn\theta} d\theta$$

$$\Rightarrow X(\theta) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a\left(\frac{\theta + 2k\pi}{T}\right)$$

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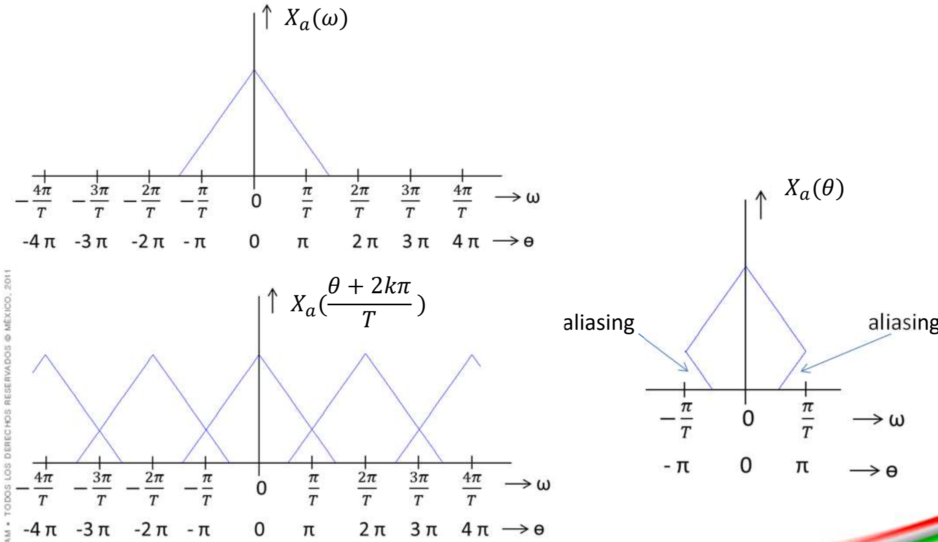
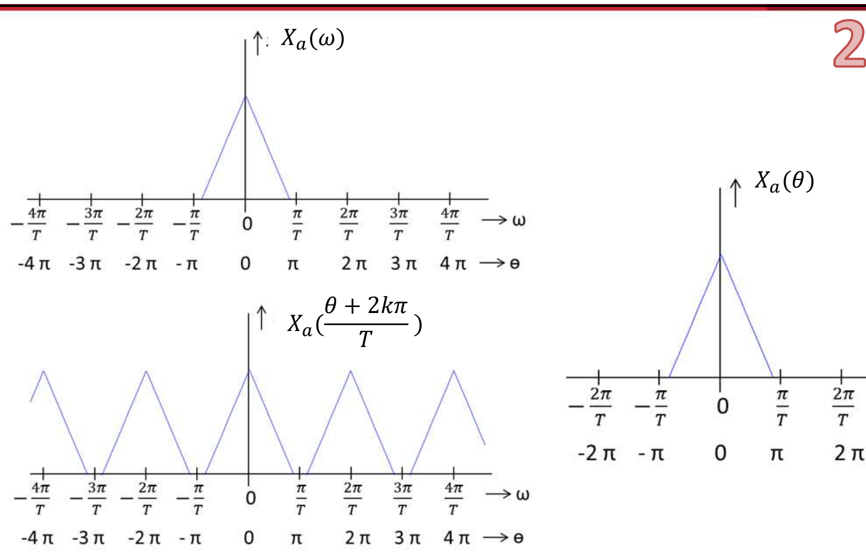
2.3

$$X(\theta)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_a\left(\frac{\theta+2k\pi}{T}\right)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_a\left(\omega+\frac{2k\pi}{T}\right)$$



3

2.4



No aliasing if: $X_a(\omega)=0 \quad for \quad |\omega|\geq \pi/T \quad or \quad |\theta|\geq\pi$

Then: $X(\theta)=X(\omega T)=\frac{1}{T}X_a(\omega) \quad for \quad -\frac{\pi}{T}\leq\omega\leq\frac{\pi}{T}$
$or \quad -\pi\leq\theta\leq\pi$

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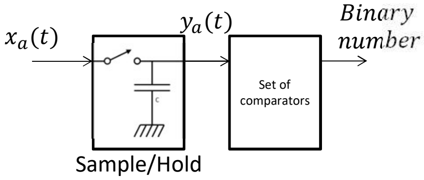
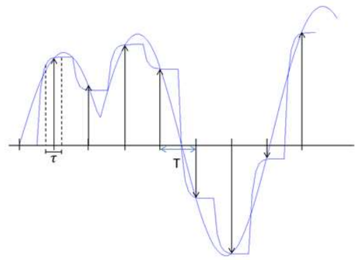
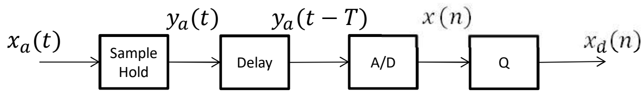
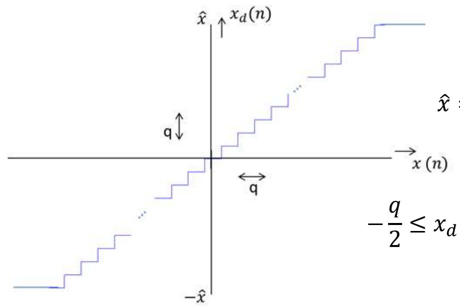
2.5

Practical A/D convertor









b-bits

$$\hat{x} = 2^{b-1}q$$

$$-\frac{q}{2} \leq x_d(n) - x(n) \leq \frac{q}{2}$$

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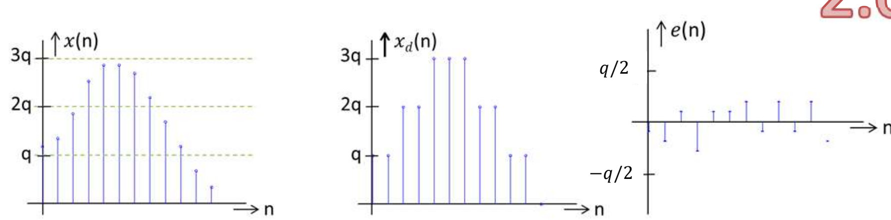
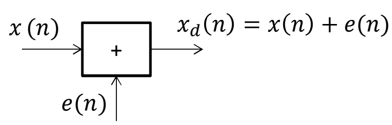
2.6

Influence of Q



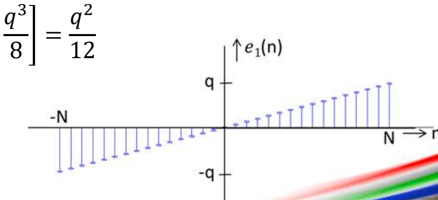
Power σ_x^2 of a digital signal:



$$\sigma_x^2 = \frac{1}{q}\int_{-q/2}^{q/2}(x-\mu)^2\,dx \qquad but \quad \mu = 0$$

$$e_1(n) = \begin{cases} qn/2N & |n| \leq N \\ 0 & |n| > N \end{cases}$$

$$\sigma_x^2 = \int_{-q/2}^{q/2}\frac{1}{q}x^2\,dx = \frac{1}{q}\frac{x^3}{3}\Bigg|_{-q/2}^{q/2} = \frac{1}{3q}\left[\frac{q^3}{8}+\frac{q^3}{8}\right] = \frac{q^2}{12}$$



Noise power: $\frac{q^2}{12}$

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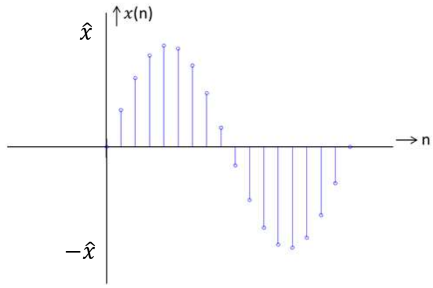
2.7

Maximum signal power for a sine wave



B bits

$$\hat{x} = 2^{B-1}q$$

$$x_{EFF} = \frac{1}{\sqrt{2}}2^{B-1}q$$

$$P_x = \frac{1}{2}2^{2B-2}q^2 = \frac{2^{2B}q^2}{8}$$

Signal to noise ratio: $\frac{S}{N} = \frac{P_x}{P_n} = \frac{\frac{2^{2B}q^2}{8}}{\frac{q^2}{12}} = \frac{12}{8}2^{2B} = \frac{3}{2}2^{2B}$

In decibels: $\frac{S}{N} = 10log\left(\frac{3}{2}2^{2B}\right) = 10log(2^{2B}) + 10\log\left(\frac{3}{2}\right)$

$$= 20Blog(2) + 1.75 = 6B + 1.75$$

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2.8

Proof that: $\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$

$N = 1$: $\sum_{n=0}^{1} n^2 = 0^2 + 1^2 = 1 = \frac{1 \cdot 2 \cdot 3}{6}$ $\quad True$

Suppose $\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$ is true for N_1

Prove that $\sum_{n=1}^{N+1} n^2 = \frac{(N+1)(N+2)(2N+3)}{6}$

$$\sum_{n=1}^{N+1} n^2 = (N+1)^2 + \sum_{n=1}^{N} n^2 = (N+1)^2 + \frac{N(N+1)(2N+1)}{6}$$

$$= \frac{6(N+1)^2 + N(N+1)(2N+1)}{6} = \frac{(N+1)\,(6N+6+2N^2+N)}{6}$$

$$= \frac{(N+1)\,(2N^2+7N+6)}{6} = \frac{(N+1)(N+2)(2N+3)}{6}$$

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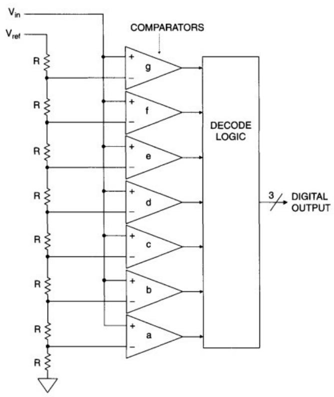
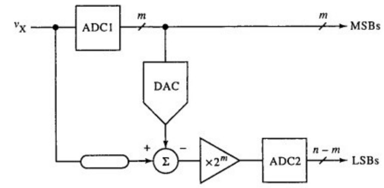
ADC Converters

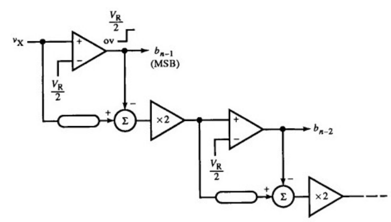
2.8b



Flash converter



Multistage converter



One bit converter

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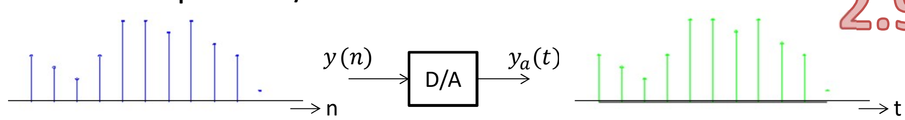
Theoretical description of a D/A convertor

2.9

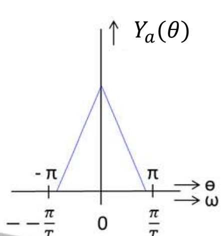


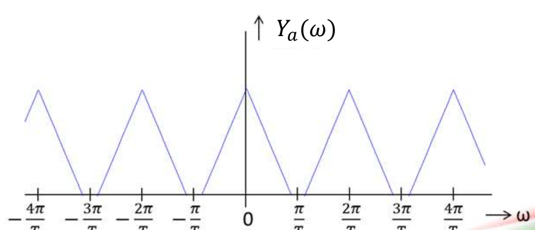
$$y_a(t) = \sum_{n=-\infty}^{\infty} y(n)\delta_a(t - nT)$$

$$Y_a(\omega) = \int_{-\infty}^{\infty} y_a(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} y(n)\delta_a(t - nT)\, e^{-j\omega t}dt$$

$$= \sum_{n=-\infty}^{\infty} y(n) \int_{-\infty}^{\infty} \delta_a(t - nT)e^{-j\omega t}dt = \sum_{n=-\infty}^{\infty} y(n)e^{-jn\omega T} = \sum_{n=-\infty}^{\infty} y(n)e^{-jn\theta} = Y(\theta)$$

Therefore: $Y_a(\omega) = Y(\theta) = Y(\omega T)$





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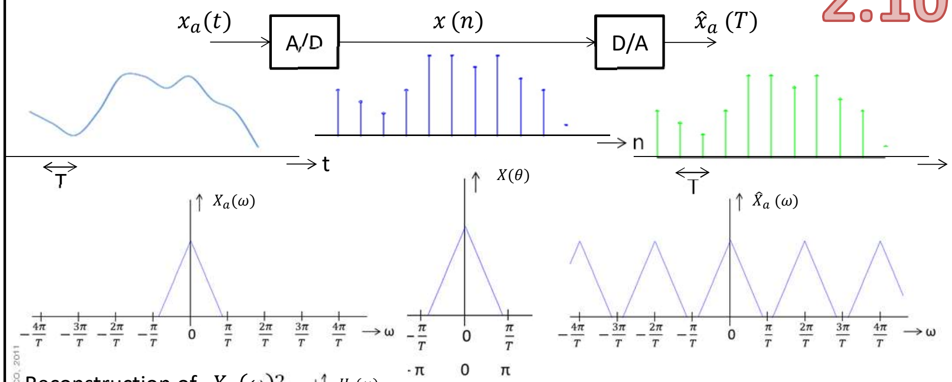
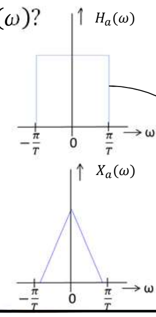
Ideal A/D and D/A process

2.10



Reconstruction of $X_a(\omega)$?

See signal analysis page 5.4 and 5.5



Reconstruction filter

$$h_a(t) = \frac{\sin(\pi t/T)}{\pi t/T}$$

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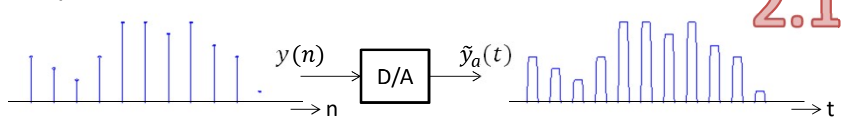
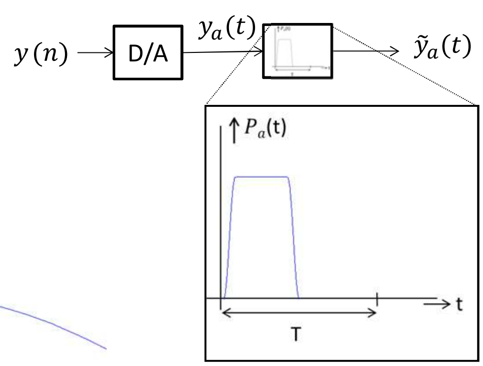
Practical D/A convertor

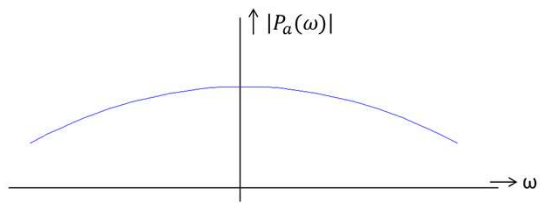
2.11



$$\tilde{y}_a(t) = \sum_{n=-\infty}^{\infty} y(n) P_a(t - nT)$$



$$\tilde{Y}_a(\omega) = Y_a(\omega) P_a(\omega)$$



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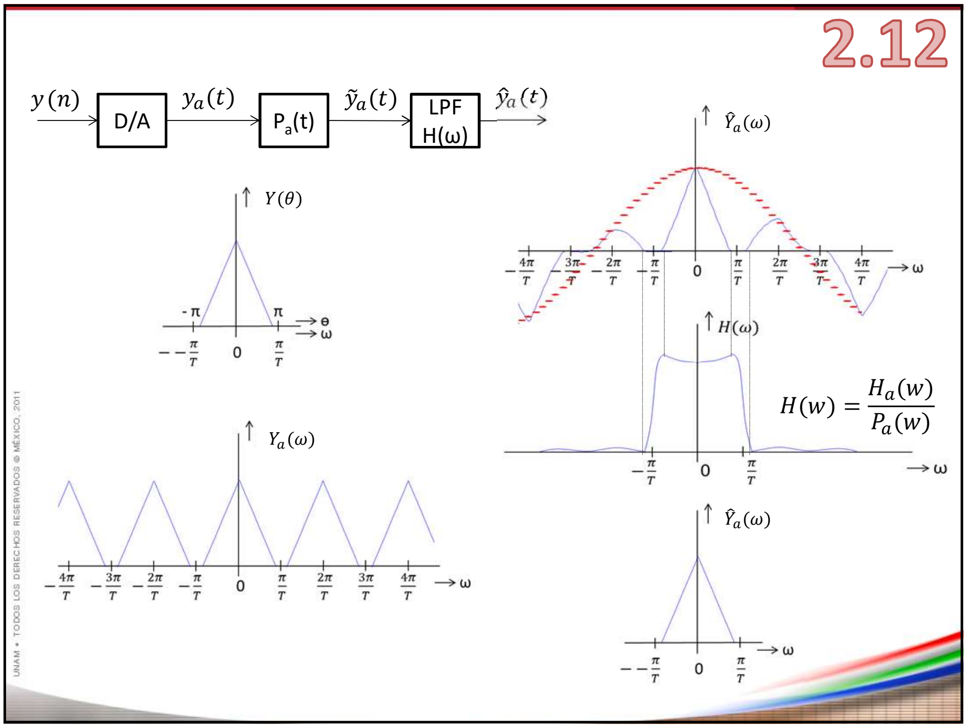
2.12
y(n)
D/A
y_a(t)
P_a(t)
ỹ_a(t)
LPF
H(ω)
ŷ_a(t)
Y(θ)
Y_a(ω)
Ŷ_a(ω)
H(ω)
H(w) = H_a(w) / P_a(w)
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