V - 1. a) Sketch the time-function that is specified by the Bartlett window for $N = 15$.

b) Compute the corresponding spectrum.

c) Which phase function $\varphi(\theta)$ must be introduced in the transmission function

$$H_d(\theta) = \begin{cases} e^{j\varphi(\theta)} & |\theta| < \frac{\pi}{10} \\ 0 & \text{elsewhere} \end{cases}$$

to obtain a linear phase FIR filter after applying the window of a).

d) Compute the filter coefficients of the FIR filter thus obtained.

V - 2. An ideal high-pass filter has the transmission function:

$$H_d(\theta) = \begin{cases} 0 & |\theta| < \frac{8\pi}{9} \\ e^{j\varphi(\theta)} & \frac{8\pi}{9} < |\theta| < \pi \end{cases}$$

We want to find the impulse response of an FIR filter with length $N = 27$ by using frequency sampling. To this end we set $\varphi(\theta) = -13\,\theta$, and define

$$\bar{H}_k = H_d\left(\frac{2\pi}{27}k\right). \qquad k = 0, \ldots, 26.$$

This definition does not specify $\bar{H}_{12}$ and $\bar{H}_{15}$.

a) Determine the impulse response of the FIR filter for the following two cases:

i) $\bar{H}_{12} = \bar{H}_{15} = 0$

ii) $\bar{H}_k = \frac{1}{2} e^{j\varphi(\theta_k)}$ $\qquad k = 12, 15$

$\qquad \theta_k = \frac{2\pi}{27} \cdot k.$

b) For both cases compute the transmission function $H(\theta)$ for $\theta = \pi$.

V-3. Assume that the following integral has to be approximated on a digital computer:

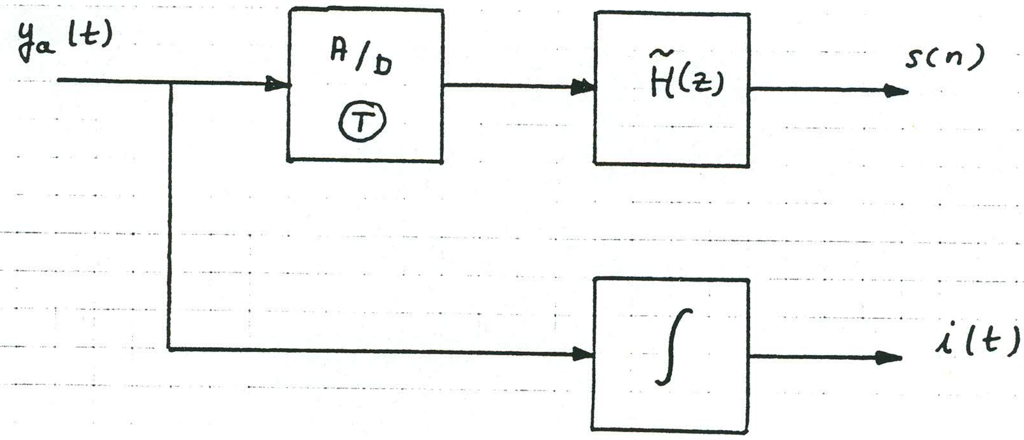
$$i(t) = \int_0^t y_a(\tau)\, d\tau .$$

In the integration routine the integration is performed by dividing the interval $(0,t)$ in subintervals of length T and computing the sum

$$s(n) = T.\sum_{k=1}^{n} \left\{ \frac{y_a(kT) + y_a((k-1)T)}{2} \right\}$$

where n is such that $t = nT$. (This corresponds to the trapezoidal rule for numerical integration).

a) Show that this integration routine can be modelled by the digital system shown below, and determine the system function $\tilde{H}(z)$.



b) The above model suggests a p-plane to z-plane transformation of the form:

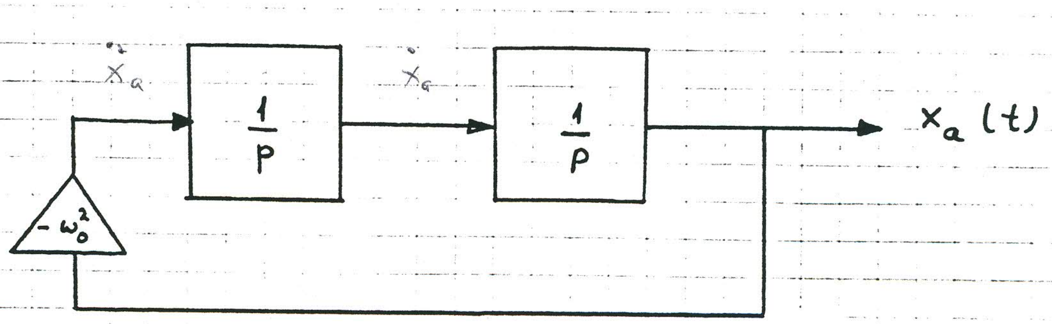
$$\frac{1}{p} \longrightarrow \tilde{H}(z)$$

How does this transform map the $j\omega$-axis of the p-plane?

c) An analog oscillator satisfies the differential equation

$$\ddot{x}_a(t) + \omega_0^2\, x(t) = 0$$

and can thus be implemented as shown below.



Applying the transform of b) a digital system will be obtained. Determine the structure of this system.
Will it be a digital oscillator?
If so, what will be the frequency of the oscillation?