

Restauración óptima de imágenes

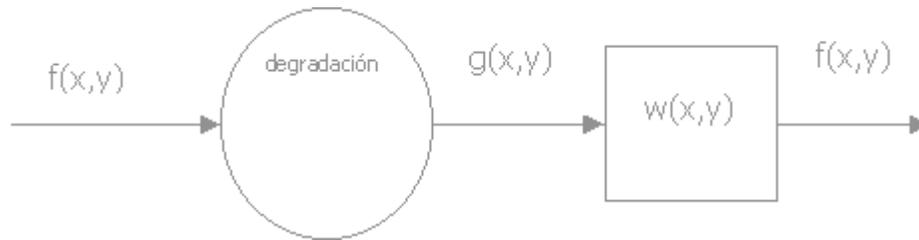


Imagen similar a la imagen original:

$$\hat{f}(x,y)$$

Modelo algebraico de procesamiento de imágenes.

Caso 1D.



$$g(x) = \sum_{i=-\infty}^{\infty} h(x-i) f(i)$$

a) Suponer que h y f son finitos y de longitud N.

$$g(x) = \sum_{i=0}^{N-1} h(x-i) f(i)$$

Se puede decir que es una convolución circular.

Agrego ceros a h y f para que g sea una convolución lineal.

$$\bar{g} = \bar{H} \bar{f} \quad \text{Señales} \longrightarrow \text{Vectores}$$

$$g(x) \longrightarrow \bar{g} = \begin{bmatrix} g(0) \\ g(1) \\ \vdots \\ g(N-1) \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{(N-1)} \end{bmatrix}$$

$$f(x) \longrightarrow \bar{f} = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{(N-1)} \end{bmatrix}$$

$$H = \begin{bmatrix} & \\ & \\ & \\ & \end{bmatrix}$$

$$\begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{(N-1)} \end{bmatrix} = \begin{bmatrix} h_0 & h_{-1} & & h_{1-N} \\ h_1 & h_0 & & h_{2-N} \\ & & & \\ & & & \\ h_{N-1} & h_{N-2} & & h_0 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{(N-1)} \end{bmatrix}$$

Sustituyo en:

$$g(x) = \sum_{i=0}^{N-1} h(x-i) f(i)$$

Matriz circulante. Cada renglón lo puedo construir por el renglón anterior, igual en las columnas.

Por ser una matriz circular los valores se repiten.

Caso 2D

$$g(x) = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} h(x-i, y-j) f(i,j)$$

$$g(x,y) \longrightarrow \bar{g}$$

$\begin{bmatrix} g_{00} \\ g_{01} \\ \vdots \\ g_{0M-1} \\ \vdots \\ g_{10} \\ g_{11} \\ \vdots \\ g_{1M-1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ g_{N-10} \\ g_{N-11} \\ \vdots \\ g_{N-1M-1} \end{bmatrix}$	$\begin{bmatrix} h_{00} & h_{0M-1} & \dots & h_{01} & \vdots & h_{N-10} & h_{N-1M-1} & \dots & h_{N-11} & \vdots & h_{10} & h_{1M-1} & \dots & h_{11} \\ h_{01} & h_{00} & \dots & h_{02} & \vdots & h_{N-11} & h_{N-10} & \dots & h_{N-12} & \vdots & h_{11} & h_{10} & \dots & h_{12} \\ \vdots & \vdots \\ h_{0M-1} & h_{0M-2} & \dots & h_{00} & \vdots & h_{N-1M-1} & h_{N-2M-2} & \dots & h_{N-10} & \vdots & h_{1M-1} & h_{1M-2} & \dots & h_{10} \\ \vdots & \vdots \\ h_{10} & h_{1M-1} & \dots & h_{11} & \vdots & h_{00} & h_{0M-1} & \dots & h_{01} & \vdots & h_{20} & h_{2M-1} & \dots & h_{21} \\ h_{11} & h_{10} & \dots & h_{12} & \vdots & h_{01} & h_{00} & \dots & h_{02} & \vdots & h_{21} & h_{20} & \dots & h_{22} \\ \vdots & \vdots \\ h_{1M-1} & h_{1M-2} & \dots & h_{10} & \vdots & h_{0M-1} & h_{0M-2} & \dots & h_{00} & \vdots & h_{2M-1} & h_{2M-2} & \dots & h_{20} \\ \vdots & \vdots \\ \vdots & \vdots \\ h_{N-10} & h_{N-1M-1} & \dots & h_{N-11} & \vdots & h_{N-20} & h_{N-2M-1} & \dots & h_{N-21} & \vdots & h_{00} & h_{0M-1} & \dots & h_{01} \\ h_{N-11} & h_{N-10} & \dots & h_{N-12} & \vdots & h_{N-21} & h_{N-20} & \dots & h_{N-22} & \vdots & h_{01} & h_{00} & \dots & h_{02} \\ \vdots & \vdots \\ h_{N-1M-1} & h_{N-1M-2} & \dots & h_{N-10} & \vdots & h_{N-2M-1} & h_{N-2M-2} & \dots & h_{N-20} & \vdots & h_{0M-1} & h_{0M-2} & \dots & h_{00} \end{bmatrix}$	$\begin{bmatrix} f_{00} \\ f_{01} \\ \vdots \\ f_{0M-1} \\ \vdots \\ f_{10} \\ f_{11} \\ \vdots \\ f_{1M-1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ f_{N-10} \\ f_{N-11} \\ \vdots \\ f_{N-1M-1} \end{bmatrix}$
$NM * 1$	$NM * NM$	$NM * 1$

$$\bar{g} = \bar{H} \bar{f}$$