# Interest Points and Corners

Read Szeliski 4.1

Computer Vision

James Hays

Slides from Rick Szeliski, Svetlana Lazebnik, Derek Hoiem and Grauman&Leibe 2008 AAAI Tutorial

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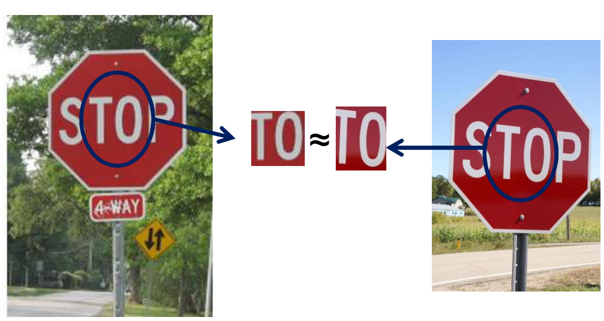
## Correspondence across views

- Correspondence: matching points, patches, edges, or regions across images

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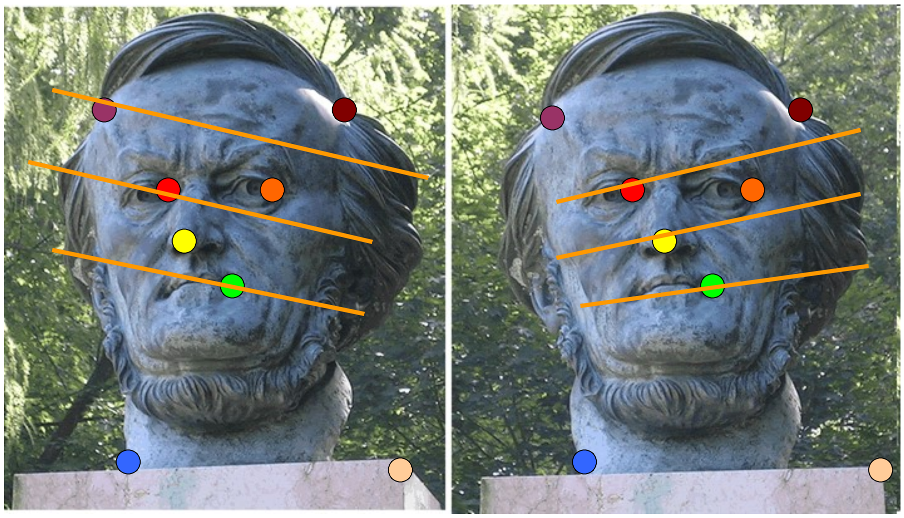
## Example: estimating "fundamental matrix" that corresponds two views



Slide from Silvio Savarese

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## Example: structure from motion

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# Applications

- Feature points are used for:
  - Image alignment
  - 3D reconstruction
  - Motion tracking
  - Robot navigation
  - Indexing and database retrieval
  - Object recognition

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# This class: interest points (continued) and local features

- Note: “interest points” = “keypoints”, also sometimes called “features”

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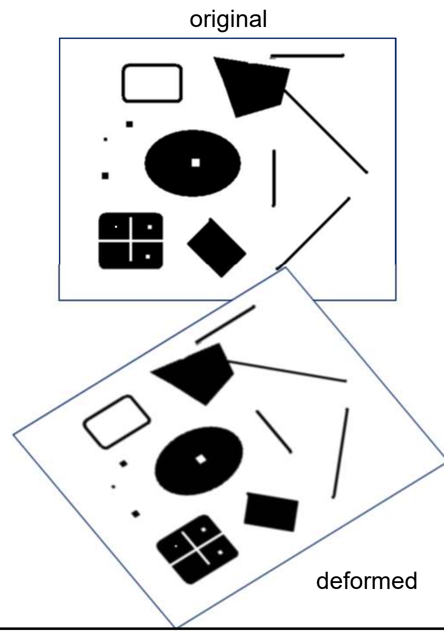
# This class: interest points

- Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.
  - Which points would you choose?

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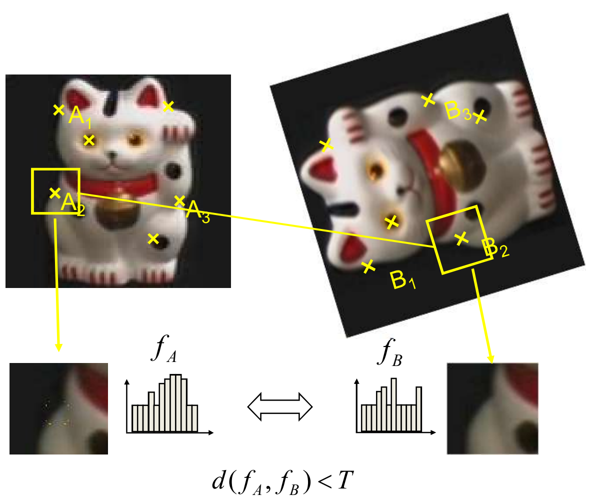
# Overview of Keypoint Matching




**1. Find a set of distinctive key-points**

**2. Define a region around each keypoint**

**3. Compute a local descriptor from the normalized region**

**4. Match local descriptors**

K. Grauman, B. Leibe

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# Goals for Keypoints

Detect points that are *repeatable* and *distinctive*

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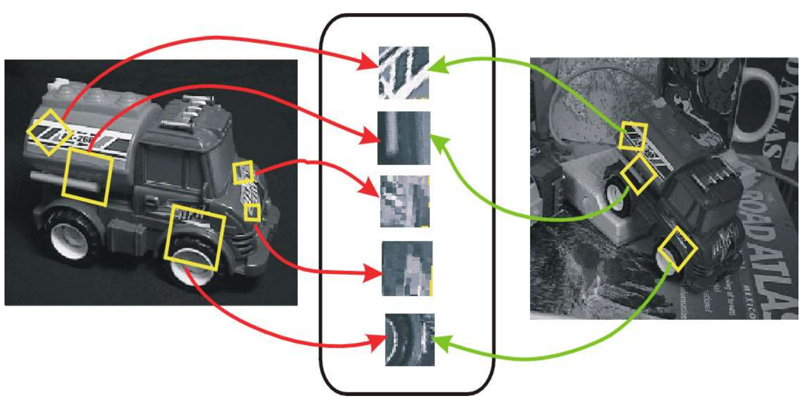
## Invariant Local Features

Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



**Features Descriptors**

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## Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

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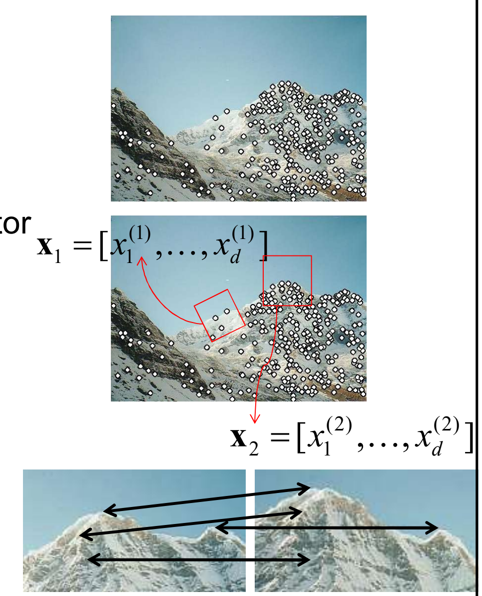
## Local features: main components

1) **Detection:** Identify the interest points
2) **Description:** Extract vector feature descriptor surrounding each interest point.
3) **Matching:** Determine correspondence between descriptors in two views

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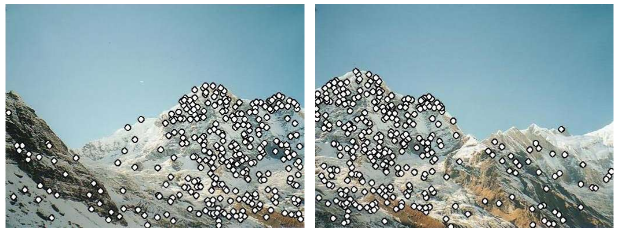
## Characteristics of good features



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

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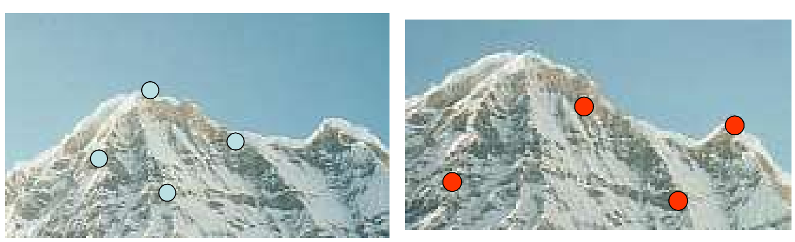
## Goal: interest operator repeatability

- We want to detect (at least some of) the same points in both images.



No chance to find true matches!

- Yet we have to be able to run the detection procedure *independently* per image.

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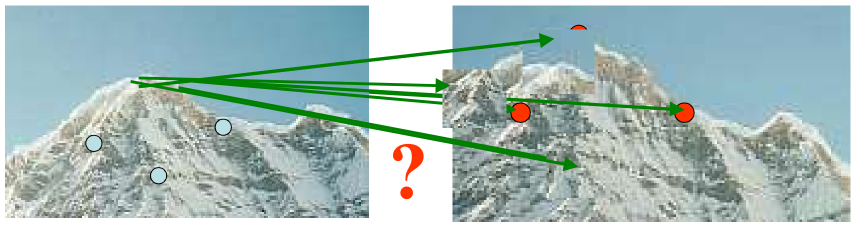
## Goal: descriptor distinctiveness

- We want to be able to reliably determine which point goes with which.




- Must provide some invariance to geometric and photometric differences between the two views.

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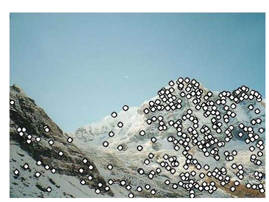
## Local features: main components

1) Detection: Identify the interest points



2) Description:Extract vector feature descriptor surrounding each interest point.

3) Matching: Determine correspondence between descriptors in two views

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# Many Existing Detectors Available

| | |
|---|---|
| Hessian & Harris | [Beaudet '78], [Harris '88] |
| Laplacian, DoG | [Lindeberg '98], [Lowe 1999] |
| Harris-/Hessian-Laplace | [Mikolajczyk & Schmid '01] |
| Harris-/Hessian-Affine | [Mikolajczyk & Schmid '04] |
| EBR and IBR | [Tuytelaars & Van Gool '04] |
| MSER | [Matas '02] |
| Salient Regions | [Kadir & Brady '01] |
| Others… | |

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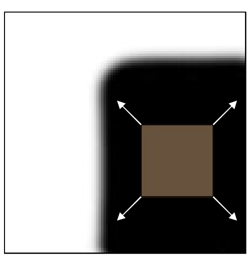
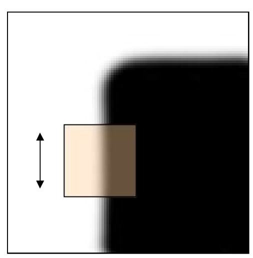
## Corner Detection: Basic Idea

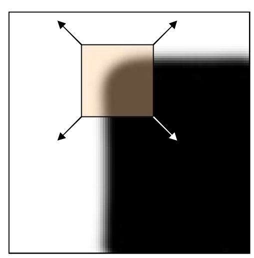
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region: no change in all directions



"edge": no change along the edge direction



"corner": significant change in all directions

Source: A. Efros

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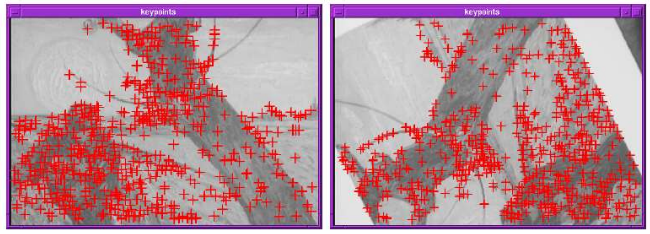
## Finding Corners



- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*: pages 147--151.

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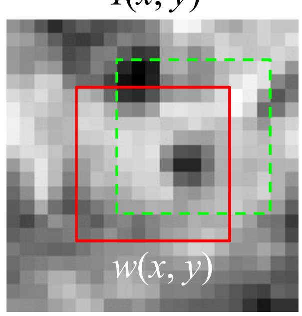
## Corner Detection: Mathematics

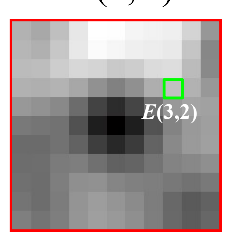
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

$I(x, y)$




$E(u, v)$

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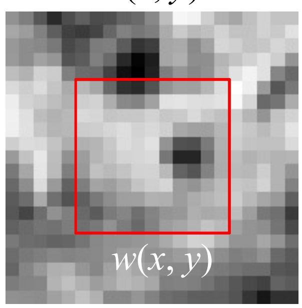
## Corner Detection: Mathematics

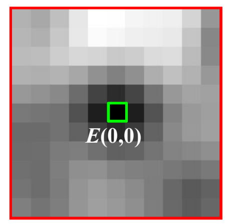
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

$I(x, y)$




$E(u, v)$

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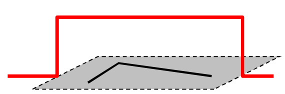
## Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

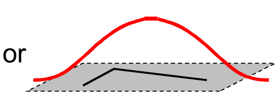
$$E(u,v) = \sum_{x,y} \underbrace{w(x,y)}_{\text{Window function}}\left[\underbrace{I(x+u,y+v)}_{\text{Shifted intensity}} - \underbrace{I(x,y)}_{\text{Intensity}}\right]^2$$

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Source: R. Szeliski

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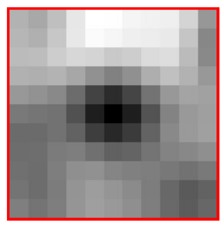
## Corner Detection: Mathematics

Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



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## Corner Detection: Mathematics

Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

But this is very slow to compute naively.
O(window_width$^2$ * shift_range$^2$ * image_width$^2$)

O( $11^2$ * $11^2$ * $600^2$ ) = 5.2 billion of these
14.6 thousand per pixel in your image

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## Corner Detection: Mathematics

Change in appearance of window *w*(*x*,*y*)
for the shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

Recall Taylor series expansion. A function f can be approximated at point a as

$$f(a)+\frac {f'(a)}{1!} (x-a)+ \frac{f''(a)}{2!} (x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+ \cdots.$$

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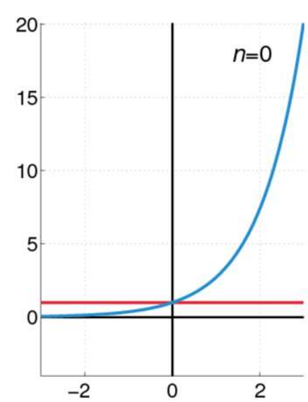
## Recall: Taylor series expansion

A function f can be approximated as

$$f(a)+\frac {f'(a)}{1!} (x-a)+ \frac{f''(a)}{2!} (x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+ \cdots.$$

Approximation of
f(x) = $e^x$
centered at f(0)

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# Taylor Series for 2D Functions

$$f(x+u,y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y) +$$

**First partial derivatives**

$$\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uv f_{xy} x,y + v^2 f_{yy}(x,y)\right] +$$

**Second partial derivatives**

$$\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

**Third partial derivatives**

$+ \ldots$ (Higher order terms)

First order approx

$$f(x+u,y+v) \approx f(x,y) + uf_x(x,y) + vf_y(x,y)$$

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# Harris Corner Derivation

$$\sum [I(x+u,y+v) - I(x,y)]^2$$

$$\approx \sum [I(x,y) + uI_x + vI_y - I(x,y)]^2$$ First order approx

$$= \sum u^2 I_x^2 + 2uvI_xI_y + v^2 I_y^2$$

$$= \sum \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$ Rewrite as matrix equation

$$= \begin{bmatrix} u & v \end{bmatrix} \left( \sum \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \right) \begin{bmatrix} u \\ v \end{bmatrix}$$

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# Harris Detector: Mathematics

For small shifts $[u,v]$ we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \quad M \begin{bmatrix} u \\ v \end{bmatrix}$$

where $M$ is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

**Windowing function - computing a weighted sum (simplest case, w=1)**

**Note: these are just products of components of the gradient, Ix, Iy**

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# Intuitive Way to Understand Harris

Treat gradient vectors as a set of (dx,dy) points with a center of mass defined as being at (0,0).

Fit an ellipse to that set of points via scatter matrix

Analyze ellipse parameters for varying cases…

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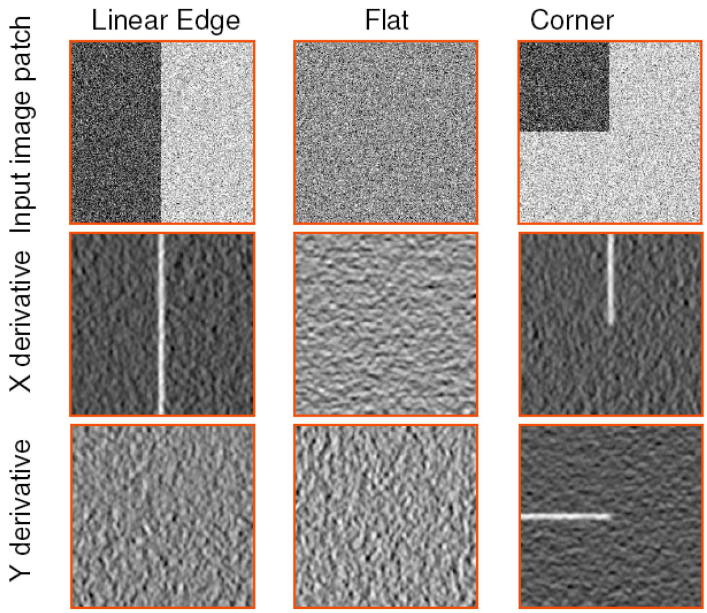
# Example: Cases and 2D Derivatives

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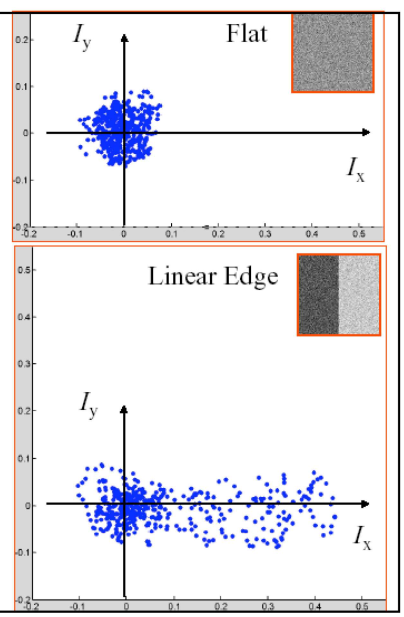
# Plotting Derivatives as 2D Points

The distribution of the $x$ and $y$ derivatives is very different for all three types of patches

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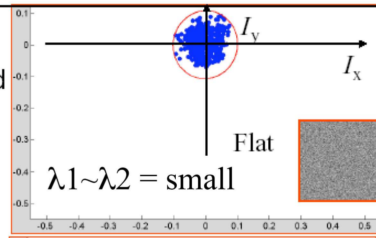
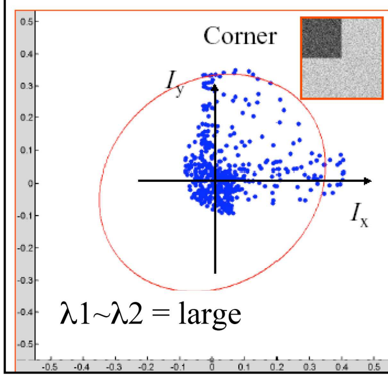
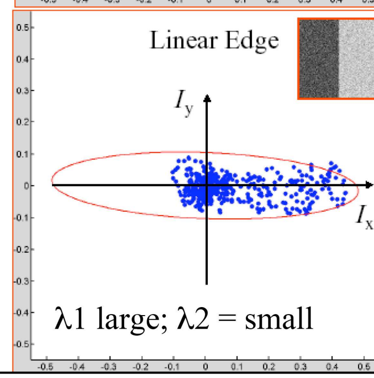
## Fitting Ellipse to each Set of Points

The distribution of $x$ and $y$ derivatives can be characterized by the shape and size of the principal component ellipse





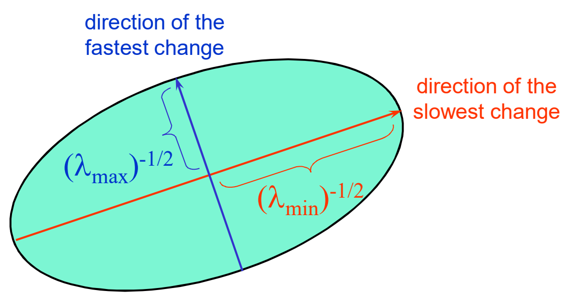

## Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

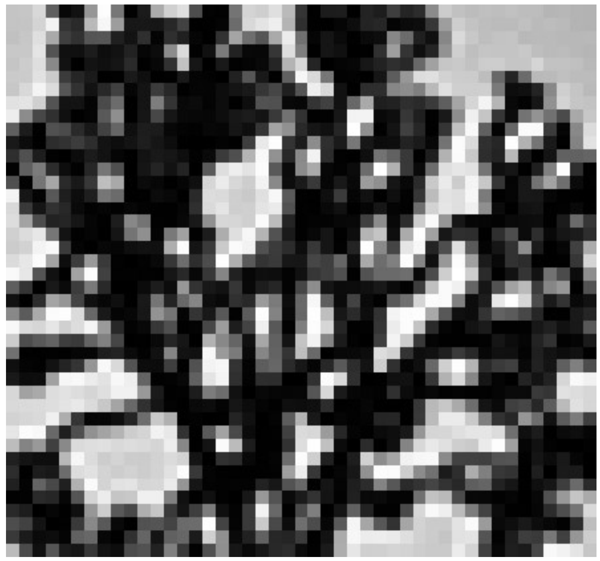
This is the equation of an ellipse.

Diagonalization of M: $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by $R$



## Visualization of second moment matrices



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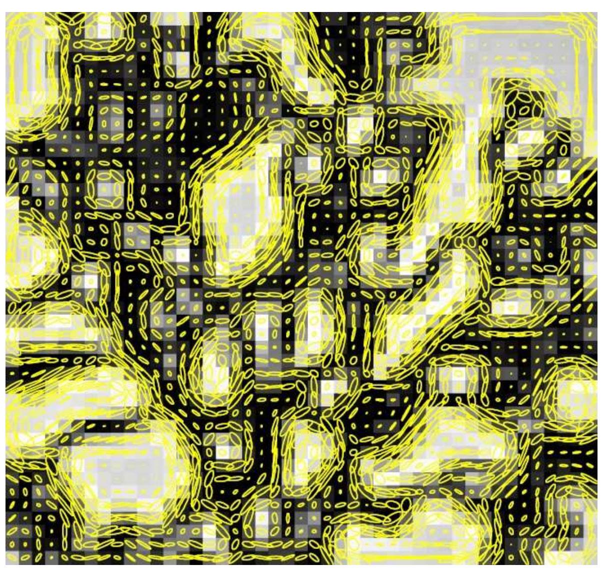
## Visualization of second moment matrices



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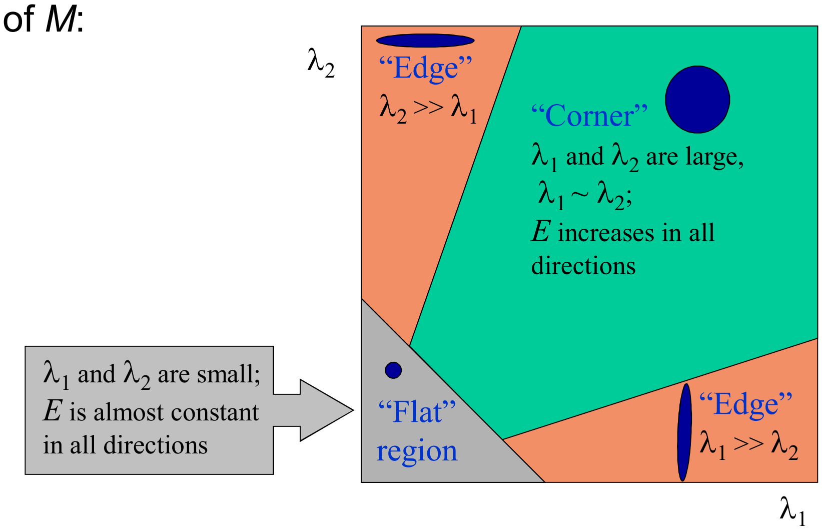
## Interpreting the eigenvalues

Classification of image points using eigenvalues of *M*:

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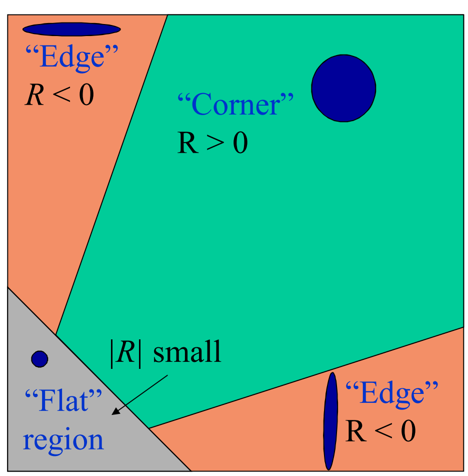
## Corner response function

$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

$\alpha$: constant (0.04 to 0.06)

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## Harris corner detector

1) Compute *M* matrix for each image window to get their *cornerness* scores.
2) Find points whose surrounding window gave large corner response (*f*> threshold)
3) Take the points of local maxima, i.e., perform non-maximum suppression

C.Harris and M.Stephens. [“A Combined Corner and Edge Detector.”]() *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

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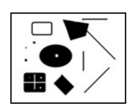
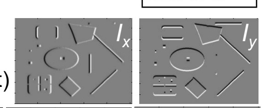
## Harris Detector [Harris88]

- Second moment matrix



$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

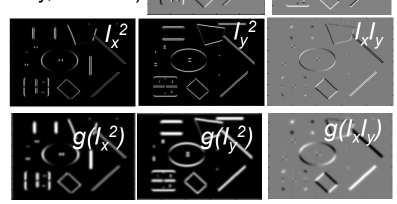
1. Image derivatives (optionally, blur first)




$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

2. Square of derivatives

3. Gaussian filter $g(\sigma_I)$




4. Cornerness function – both eigenvalues are strong

$$har = \det[\mu(\sigma_I, \sigma_D)] - \alpha[\operatorname{trace}(\mu(\sigma_I, \sigma_D))^2] =$$
$$g(I_x^2)g(I_y^2) - [g(I_xI_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

5. Non-maxima suppression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## Harris Detector: Steps

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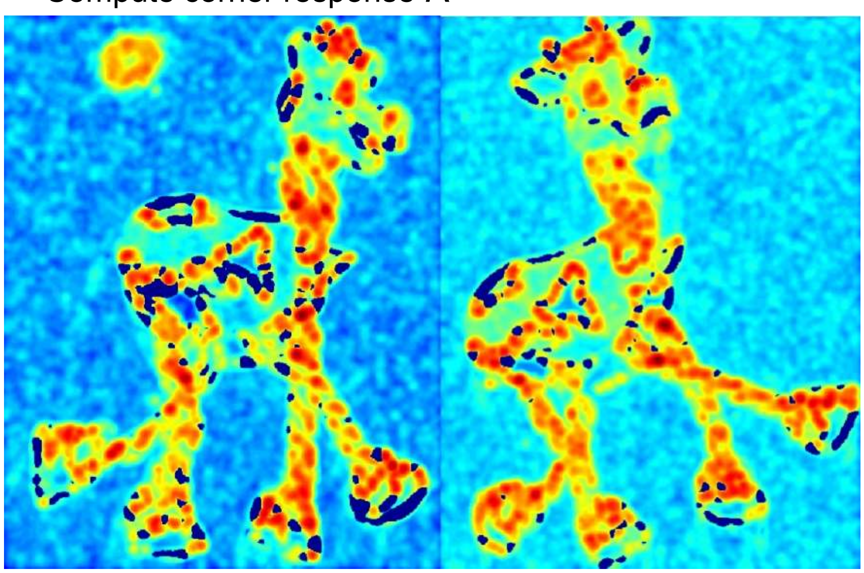
## Harris Detector: Steps

Compute corner response $R$



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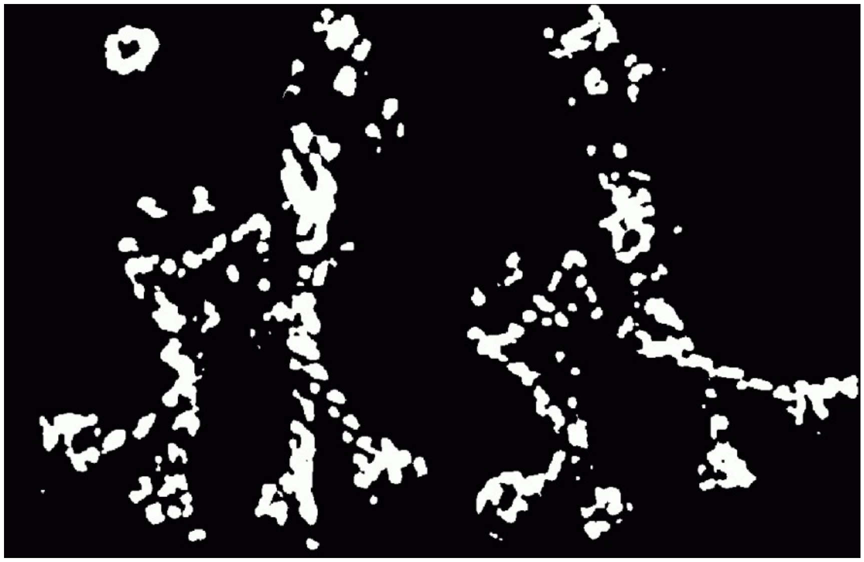
## Harris Detector: Steps

Find points with large corner response: $R$>threshold



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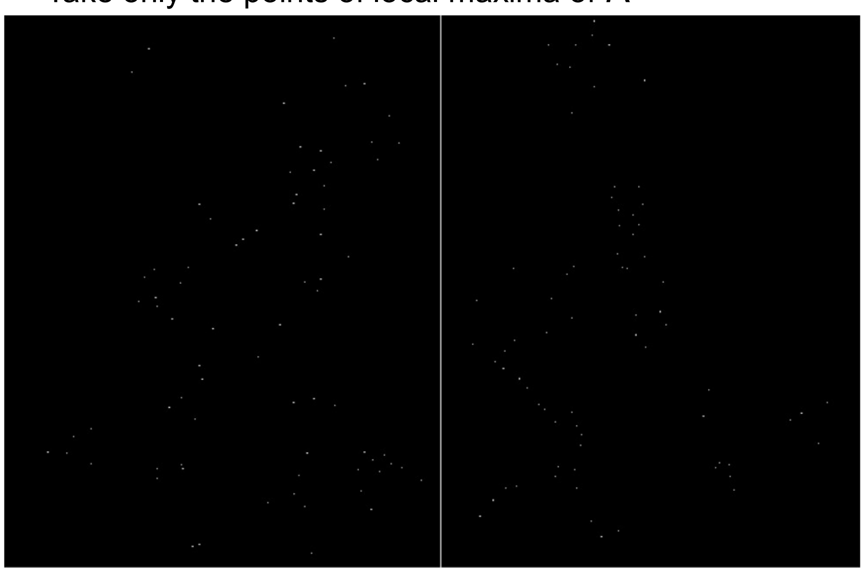
## Harris Detector: Steps

Take only the points of local maxima of $R$



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## Harris Detector: Steps

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## Invariance and covariance

- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
  - **Invariance:** image is transformed and corner locations do not change
  - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

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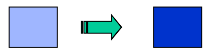
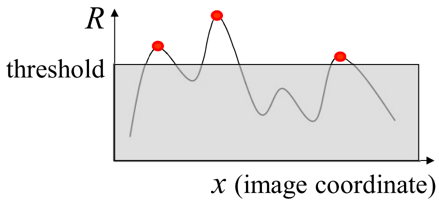
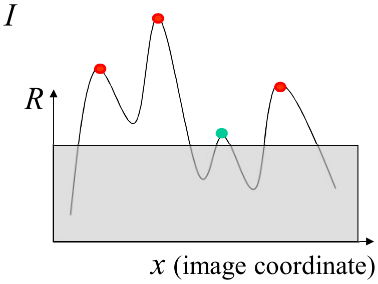
## Affine intensity change



$I \rightarrow a\,I + b$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a\,I$




*Partially invariant* to affine intensity change

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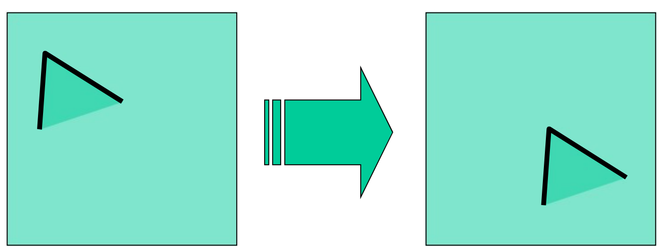
## Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

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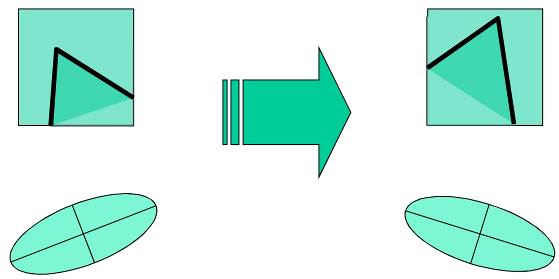
## Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

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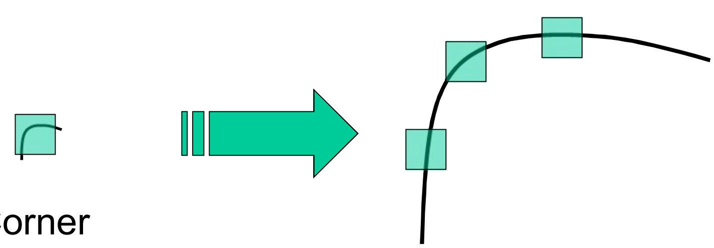
## Scaling



Corner

All points will be classified as edges

Corner location is not covariant to scaling!

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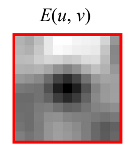
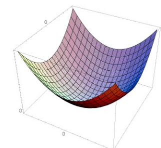
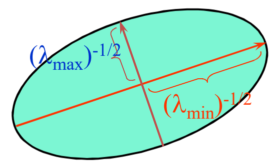
# Review: Harris corner detector




- Approximate distinctiveness by local auto-correlation.
- Approximate local auto-correlation by second moment matrix
- Quantify distinctiveness (or cornerness) as function of the eigenvalues of the second moment matrix.
- But we don't actually need to compute the eigenvalues by using the determinant and trace of the second moment matrix.






- Video chess https://youtu.be/vkWdzWeRfC4

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# So far: can localize in x-y, but not scale

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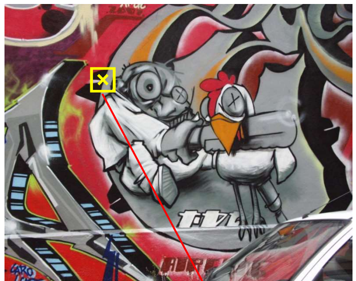
# Automatic Scale Selection



$$f(I_{i_1 \ldots i_m}(x, \sigma)) \;=\; f(I_{i_1 \ldots i_m}(x', \sigma'))$$

How to find corresponding patch sizes?

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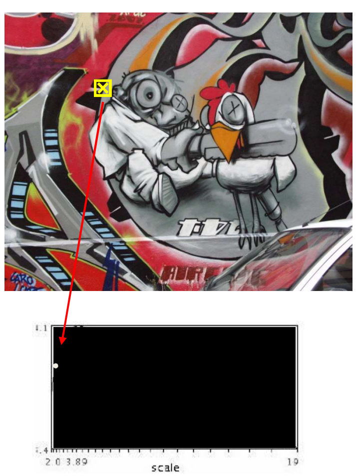
# Automatic Scale Selection

- Function responses for increasing scale (scale signature)




$f(I_{i_1 \ldots i_m}(x, \sigma))$


$f(I_{i_1 \ldots i_m}(x', \sigma))$

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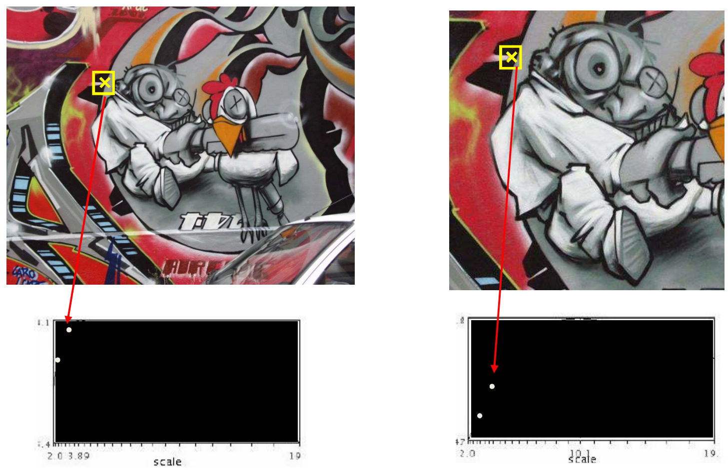
# Automatic Scale Selection

- Function responses for increasing scale (scale signature)




$f(I_{i_1 \ldots i_m}(x,\sigma))$ $f(I_{i_1 \ldots i_m}(x',\sigma))$

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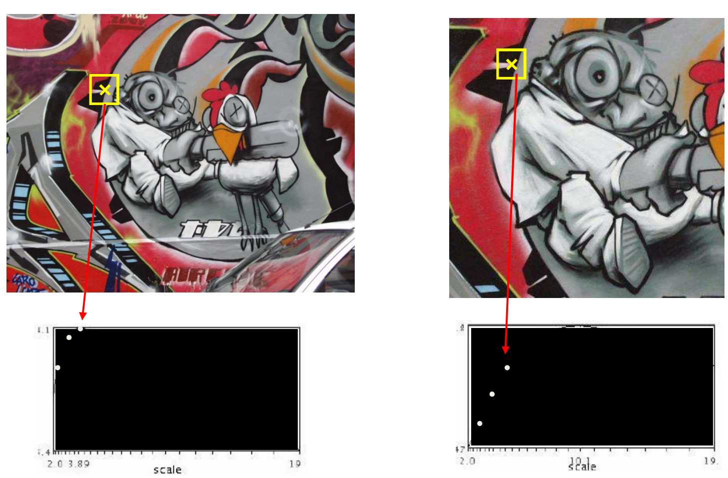
# Automatic Scale Selection

- Function responses for increasing scale (scale signature)




$f(I_{i_1 \ldots i_m}(x,\sigma))$ $f(I_{i_1 \ldots i_m}(x',\sigma))$

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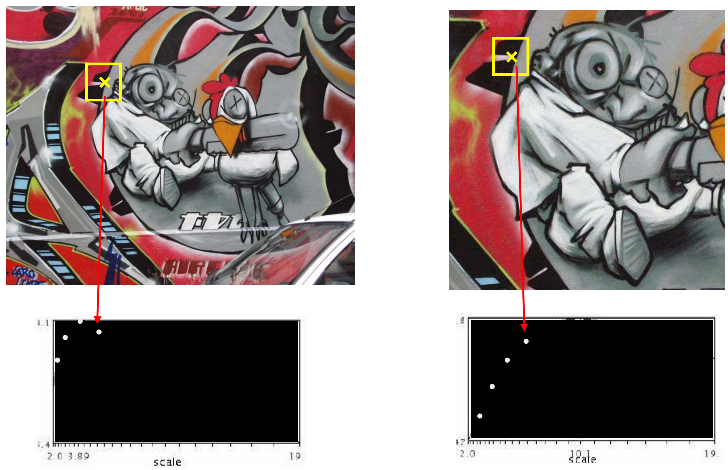
# Automatic Scale Selection

- Function responses for increasing scale (scale signature)




$f(I_{i_1 \dots i_m}(x,\sigma))$ $f(I_{i_1 \dots i_m}(x',\sigma))$

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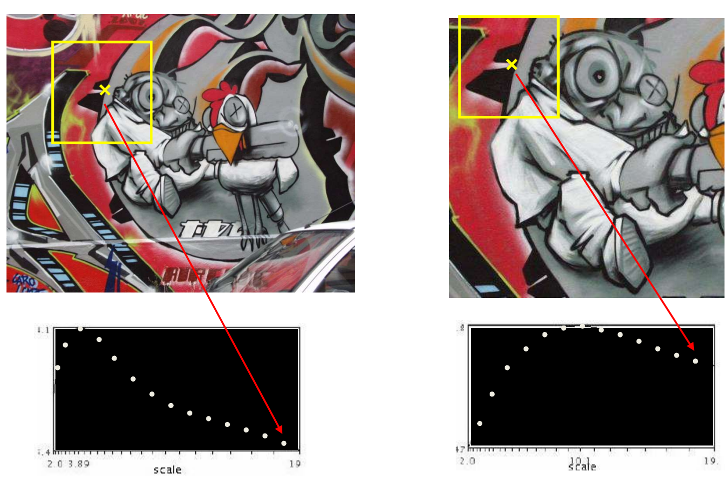
# Automatic Scale Selection

- Function responses for increasing scale (scale signature)




$f(I_{i_1 \dots i_m}(x,\sigma))$ $f(I_{i_1 \dots i_m}(x',\sigma))$

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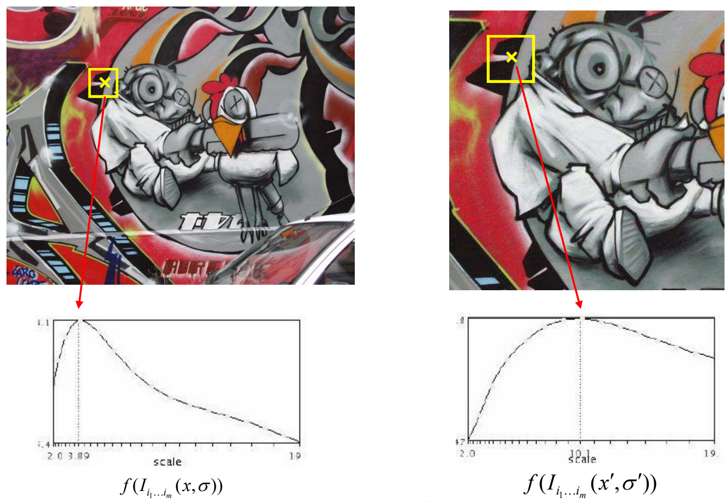
# Automatic Scale Selection

- Function responses for increasing scale (scale signature)

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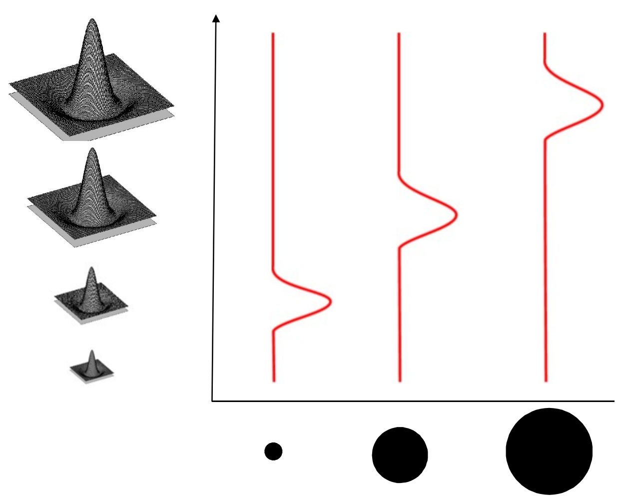
# What Is A Useful Signature Function?

- Difference-of-Gaussian = "blob" detector



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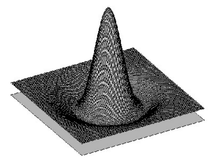
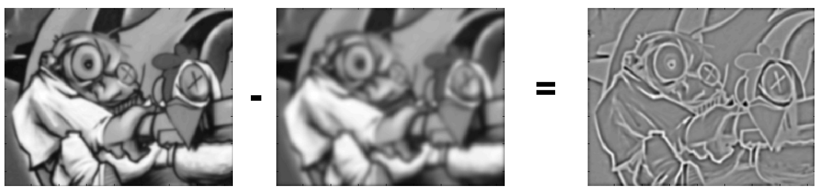
# Difference-of-Gaussian (DoG)



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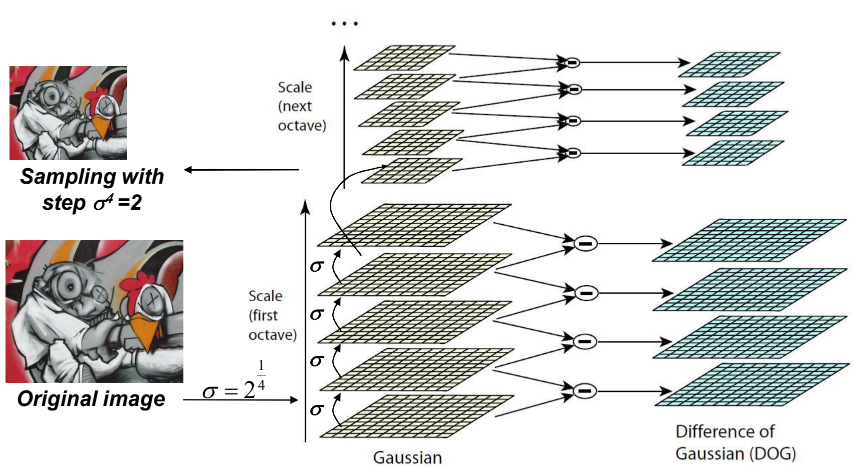
# DoG – Efficient Computation

- Computation in Gaussian scale pyramid

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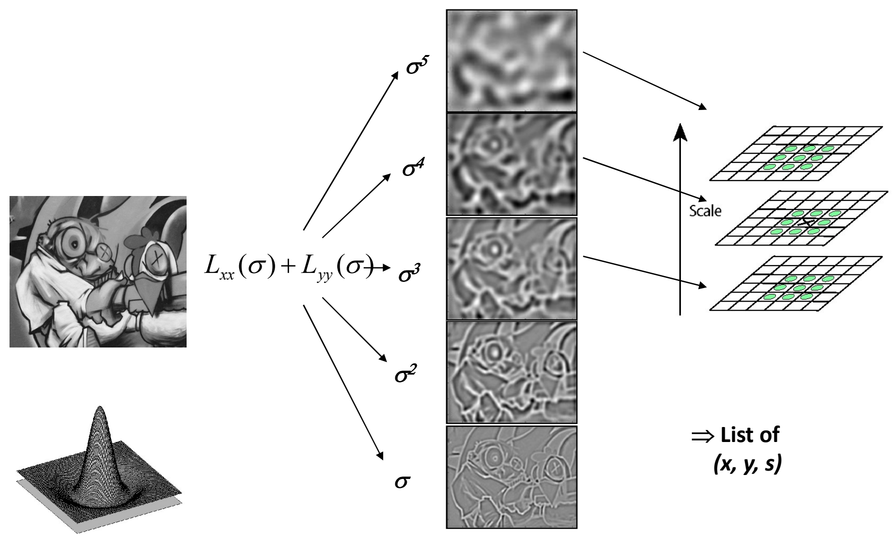
# Find local maxima in position-scale space of Difference-of-Gaussian

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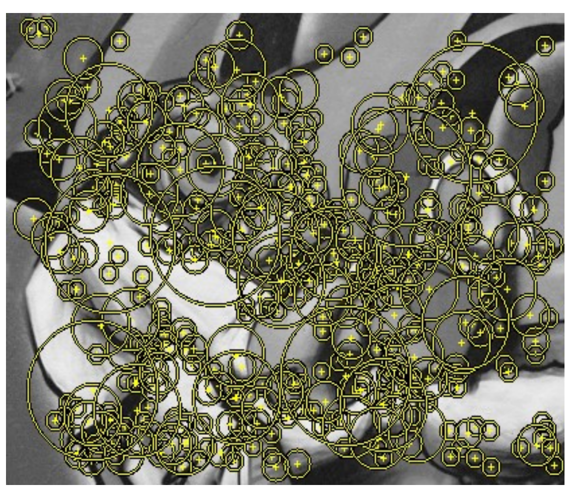
# Results: Difference-of-Gaussian



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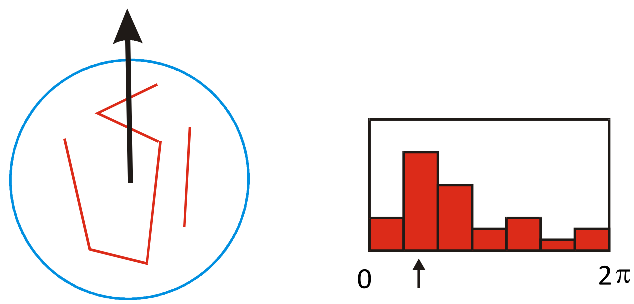
# Orientation Normalization

- Compute orientation histogram
- Select dominant orientation
- Normalize: rotate to fixed orientation

[Lowe, SIFT, 1999]

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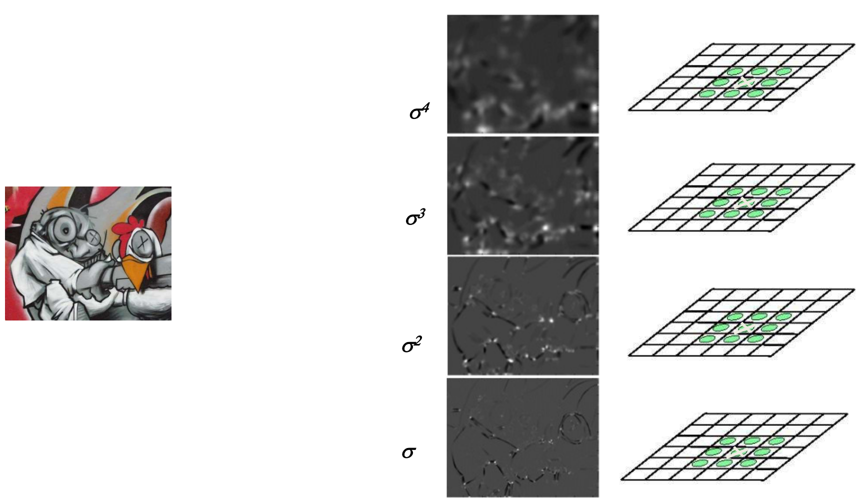
# Harris-Laplace [Mikolajczyk '01]

1. Initialization: Multiscale Harris corner detection




**Computing Harris function** **Detecting local maxima**

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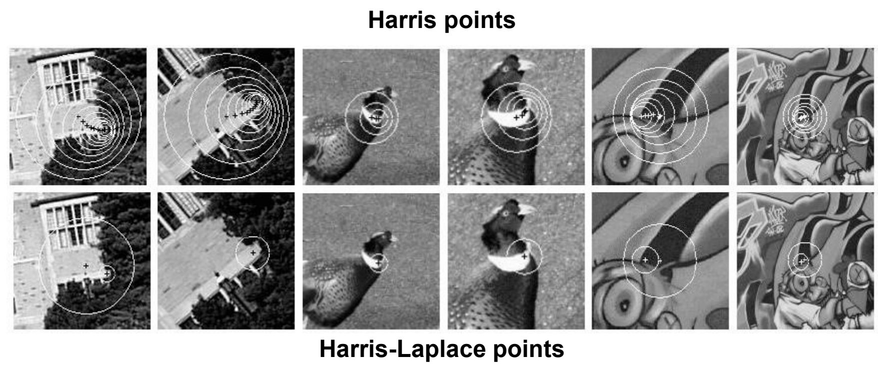
## Harris-Laplace [Mikolajczyk '01]

1. Initialization: Multiscale Harris corner detection
2. Scale selection based on Laplacian
   (same procedure with Hessian $\Rightarrow$ Hessian-Laplace)

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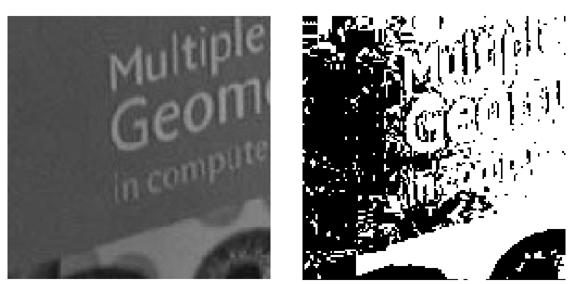
## Maximally Stable Extremal Regions [Matas '02]

- Based on Watershed segmentation algorithm
- Select regions that stay stable over a large parameter range

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# Example Results: MSER




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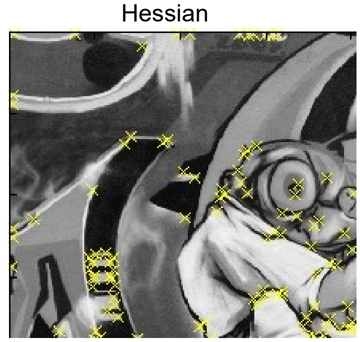
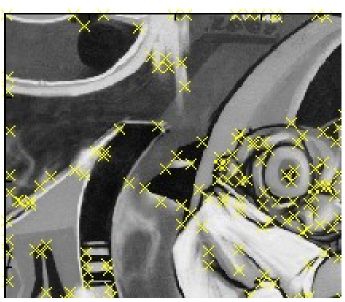
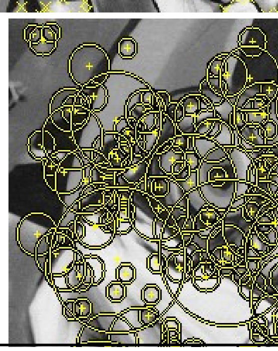
# Comparison

Hessian



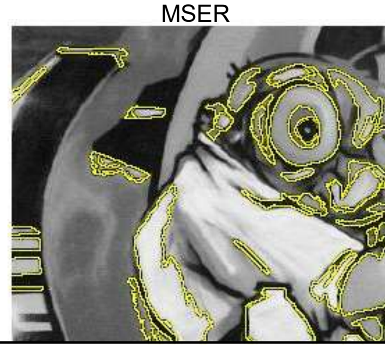


Harris



LoG

MSER



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# Available at a web site near you…

- For most local feature detectors, executables are available online:
  - http://www.robots.ox.ac.uk/~vgg/research/affine
  - http://www.cs.ubc.ca/~lowe/keypoints/
  - http://www.vision.ee.ethz.ch/~surf

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