What is an image?

- Ideally, we think of an image as a 2-dimensional light intensity function, $f(x,y)$, where $x$ and $y$ are spatial coordinates, and $f$ at $(x,y)$ is related to the brightness or color of the image at that point.
- In practice, most images are defined over a rectangle.
- Continuous in amplitude ("continuous-tone")
- Continuous in space: no pixels!

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Digital Images and Pixels

- A digital image is the representation of a continuous image $f(x,y)$ by a 2-d array of discrete samples. The amplitude of each sample is quantized to be represented by a finite number of bits.
- Each element of the 2-d array of samples is called a pixel or pel (from "picture element")
- Pixels are point samples, without extent.
- A pixel is not:
  - Round, square, or rectangular
  - An element of an image sensor
  - An element of a display
A Digital Image is Represented by Numbers

- Pixel = “picture element”
- Represents brightness at one point

A digital image can be represented as a matrix

\[
f = \begin{bmatrix}
  f(0, 0) & f(1, 0) & \cdots & f(N-1, 0) \\
  f(0, 1) & f(1, 1) & \cdots & f(N-1, 1) \\
  \vdots & \vdots & \ddots & \vdots \\
  f(0, L-1) & f(1, L-1) & \cdots & f(N-1, L-1)
\end{bmatrix}
\]

- The pixel values \(f(x, y)\) are sorted into the matrix in “natural” order, with \(x\) corresponding to the column and \(y\) to the row index. Matlab uses this convention. This results in \(f(x, y) = f_{yx}\), where \(f_{yx}\) denotes an individual element in common matrix notation.
- For a color image, \(\mathbf{f}\) might be one of the components.
Image Size and Resolution

• These images were produced by simply picking every n-th sample horizontally and vertically and replicating that value nxn times.
• We can do better
  – prefiling before subsampling to avoid aliasing
  – Smooth interpolation

Images of Different Sizes

272 pixels

280 pixels
Fewer Pixels Mean Lower Spatial Resolution

272 pixels

280 pixels

35 x 33 image interpolated to 280 x 272 pixels

Color Components

Monochrome image

\[ R(x, y) = G(x, y) = B(x, y) \]

Red \( R(x, y) \)

Green \( G(x, y) \)

Blue \( B(x, y) \)
Different numbers of gray levels

256 32 16

8 4 2

“Contouring”

How many gray levels are required?

- Contouring is most visible for a ramp

32 levels
64 levels
128 levels
256 levels

- Digital images typically are quantized to 256 gray levels.
Storage requirements for digital images

- Image LxN pixels, $2^B$ gray levels, c color components

\[
\text{Size} = L \times N \times B \times c
\]

- Example: $L=N=512$, $B=8$, $c=1$ (i.e., monochrome)
  \[
  \text{Size} = 2,097,152 \text{ bits (or 256 kByte)}
  \]

- Example: $L \times N=1024 \times 1280$, $B=8$, $c=3$ (24 bit RGB image)
  \[
  \text{Size} = 31,457,280 \text{ bits (or 3.75 MByte)}
  \]

- Much less with (lossy) compression!

Brightness discrimination experiment

- Can you see the circle?

Note: $I$ is luminance, measured in $cd/m^2$.

- Visibility threshold

\[
\frac{\Delta I}{I} \approx \text{const.} \approx 1\ldots2\%
\]

„Weber fraction“
„Weber’s Law“
Contrast with 8 Bits According to Weber’s Law

- Assume that the luminance difference between two successive representative levels is just at visibility threshold

\[
\frac{I_{\text{max}}}{I_{\text{min}}} = (1 + \text{const.})^{255}
\]

- For \( \text{const.} = 0.01 \ldots 0.02 \) \( \frac{I_{\text{max}}}{I_{\text{min}}} = 13 \ldots 156 \)

- Typical display contrast
  - Cathode ray tube 100:1
  - Print on paper 10:1

- Suggests uniform quantization in the \( \log(I) \) domain

Gamma characteristic

- Cathode ray tubes (CRT) are nonlinear

![Gamma characteristic diagram](image)

- Cameras contain \( \gamma \)-predistortion circuit

\[
U \sim I^{\frac{1}{\gamma}}
\]

\( \gamma = 2.0 \ldots 2.3 \)
log vs. $\gamma$-predistortion

![Graph showing log vs. $\gamma$-predistortion](graph.png)

- Similar enough for most practical applications

Photographic film

- Hurter & Driffield curve (H&D curve) for photographic negative

  \[ U \sim \log(I) \]

  \[ U \sim I^{\gamma} \]

  \[ \frac{I_{\text{max}}}{I_{\text{min}}} = 100 \]

- Luminance

  \[ I = I_0 \cdot 10^{-d} \]

  \[ = I_0 \cdot 10^{-d_0 \cdot \gamma} \cdot E^\gamma \]

- $\gamma$ measures film contrast
  - General purpose films: $\gamma = -0.7 \ldots -1.0$
  - High-contrast films: $\gamma = -1.5 \ldots -10$

- Lower speed films tend to have higher absolute $\gamma$