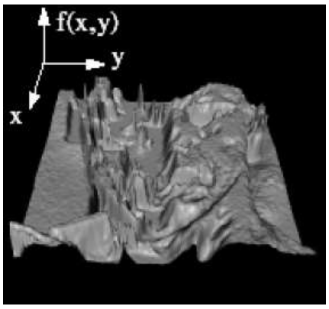
What is an image?

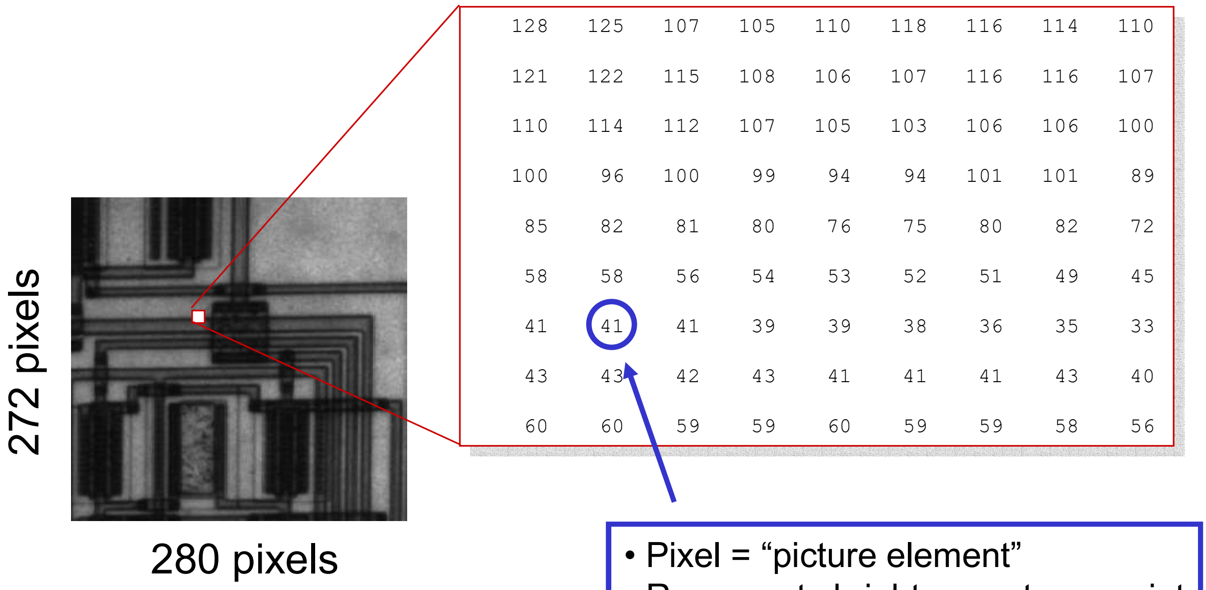
- Ideally, we think of an **image** as a 2-dimensional light intensity function, $f(x,y)$, where x and y are spatial coordinates, and f at (x,y) is related to the brightness or color of the image at that point.
- In practice, most images are defined over a rectangle.
- Continous in amplitude („continous-tone“)
- Continous in space: no pixels!



Digital Images and Pixels

- A **digital image** is the representation of a continuous image $f(x,y)$ by a 2-d array of discrete samples. The amplitude of each sample is quantized to be represented by a finite number of bits.
- Each element of the 2-d array of samples is called a **pixel** or **pel** (from „picture element“)
- Pixels are point samples, without extent.
- A pixel is <u>not</u>:
  - Round, square, or rectangular
  - An element of an image sensor
  - An element of a display

A Digital Image is Represented by Numbers



128	125	107	105	110	118	116	114	110
121	122	115	108	106	107	116	116	107
110	114	112	107	105	103	106	106	100
100	96	100	99	94	94	101	101	89
85	82	81	80	76	75	80	82	72
58	58	56	54	53	52	51	49	45
41	41	41	39	39	38	36	35	33
43	43	42	43	41	41	41	43	40
60	60	59	59	60	59	59	58	56

- Pixel = “picture element”
- Represents brightness at one point

A digital image can be represented as a matrix

$$\begin{array}{cc} \xrightarrow{\quad x \quad} & \\ \mathbf{f} = \begin{bmatrix} f(0,0) & f(1,0) & \cdots & f(N-1,0) \\ f(0,1) & f(1,1) & \cdots & f(N-1,1) \\ \vdots & \vdots & & \vdots \\ f(0,L-1) & f(1,L-1) & \cdots & f(N-1,L-1) \end{bmatrix} & \Big\downarrow y \end{array}$$

- The pixel values $f(x,y)$ are sorted into the matrix in „natural“ order, with x corresponding to the column and y to the row index. Matlab uses this convention. This results in $f(x,y) = f_{yx}$, where f_{yx} denotes an individual element in common matrix notation.
- For a color image, $\mathbf{f}$ might be one of the components.

Image Size and Resolution

200x200

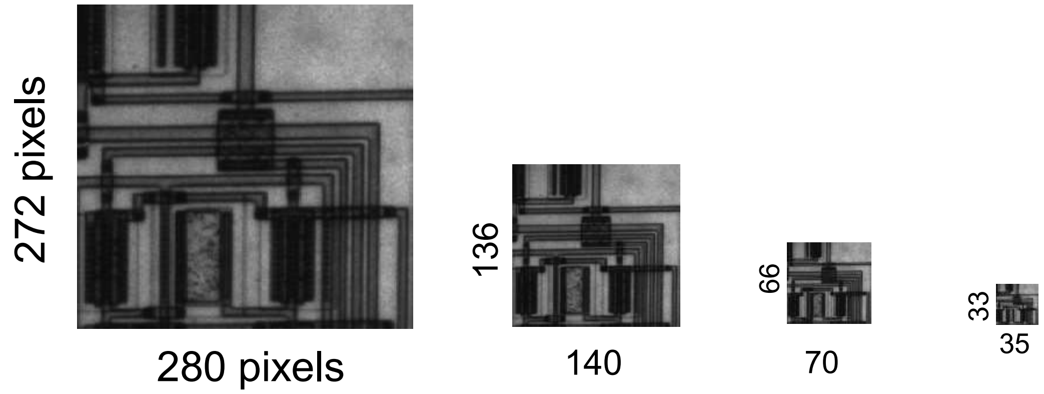
100x100

50x50

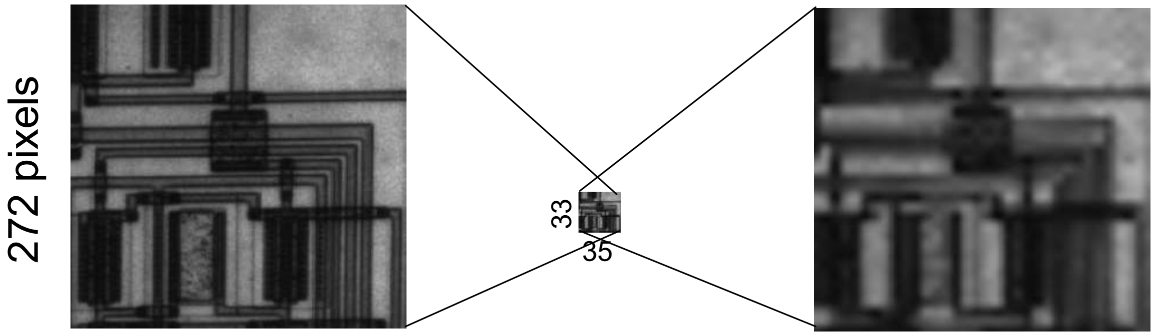
25x25

- These images were produced by simply picking every n-th sample horizontally and vertically and replicating that value nxn times.
- We can do better
  - *prefiltering before subsampling to avoid aliasing*
  - *Smooth interpolation*

Images of Different Sizes



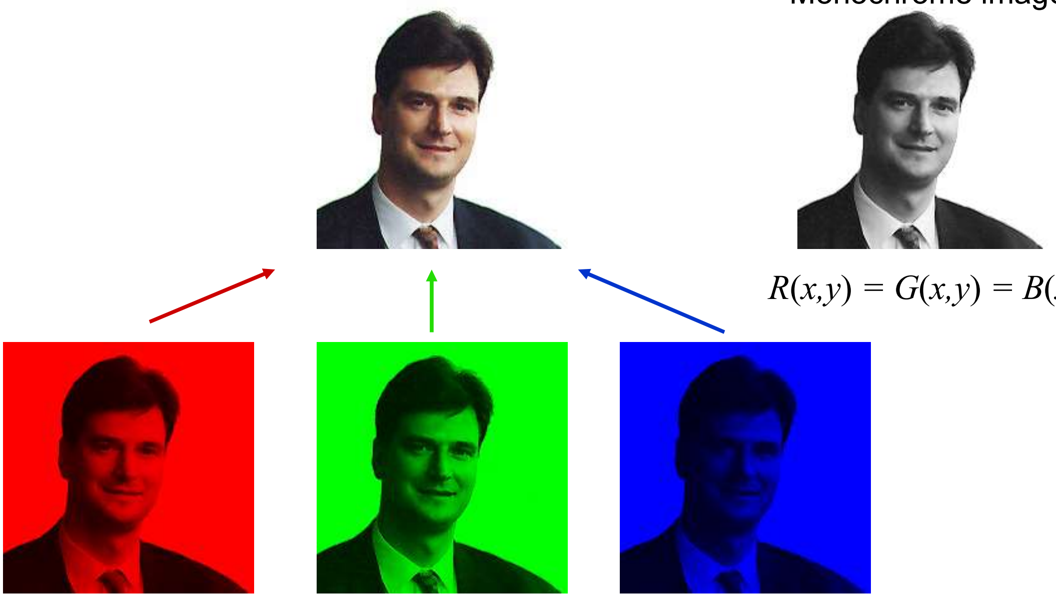
Fewer Pixels Mean Lower Spatial Resolution



280 pixels

35 x 33 image
interpolated to
280 x 272 pixels

Color Components

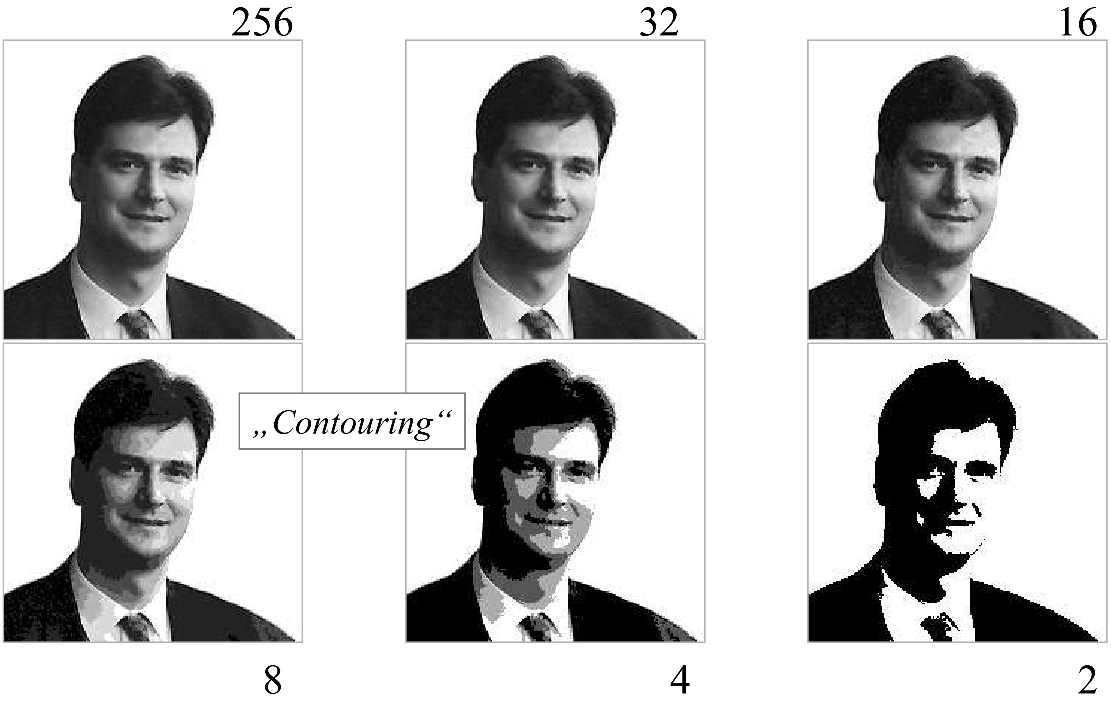


Monochrome image

$R(x,y) = G(x,y) = B(x,y)$

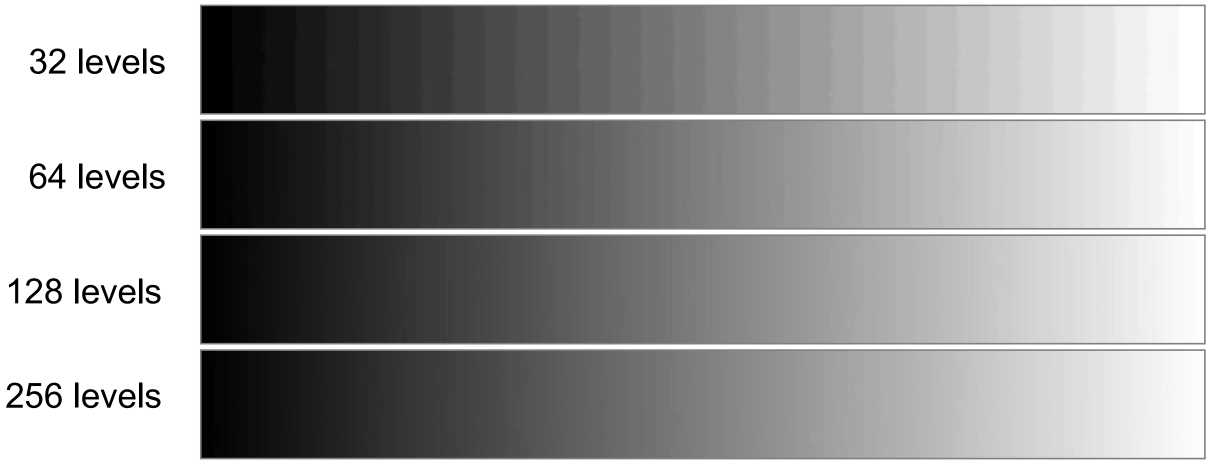
Red $R(x,y)$ Green $G(x,y)$ Blue $B(x,y)$

Different numbers of gray levels



How many gray levels are required?

- Contouring is most visible for a ramp



- Digital images typically are quantized to 256 gray levels.

Storage requirements for digital images

- Image LxN pixels, 2^B gray levels, *c* color components

$$\boxed{Size = LxNxBxc}$$

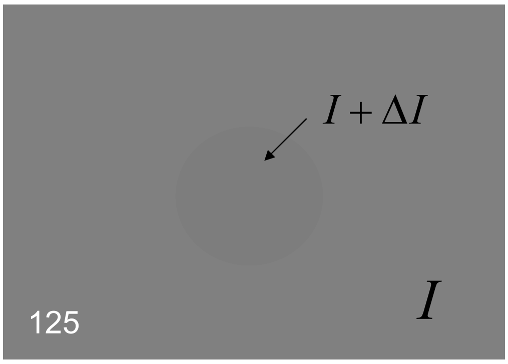
  - *Example: L=N=512, B=8, c=1 (i.e., monochrome)*
    Size = 2,097,152 bits (or 256 kByte)

  - *Example: LxN=1024x1280, B=8, c=3 (24 bit RGB image)*
    Size = 31,457,280 bits (or 3.75 MByte)

- Much less with (lossy) compression!

Brightness discrimination experiment

- Can you see the circle?



Note: I is luminance, measured in cd/m^2

- Visibility threshold

$$\boxed{\Delta I/I \approx const. \approx 1\ldots2\%}$$

„Weber fraction“
„Weber‘s Law“

Contrast with 8 Bits According to Weber‘s Law

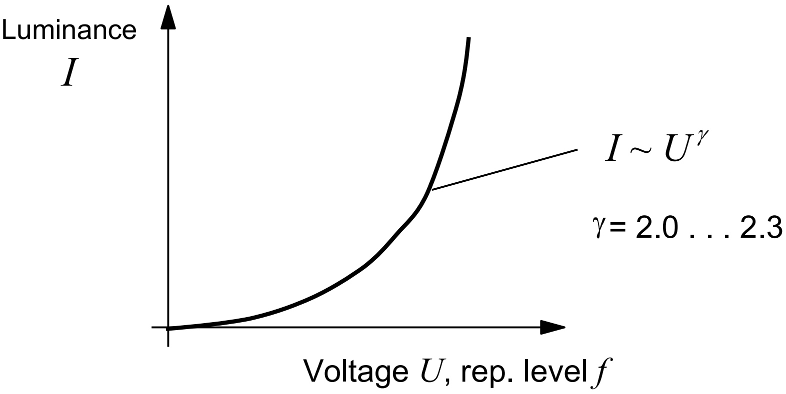
- Assume that the luminance difference between two successive representative levels is just at visibility threshold

$$\boxed{\frac{I_{\max}}{I_{\min}} = (1 + const.)^{255}}$$

- For $const. = 0.01 \cdots 0.02$ $\qquad \frac{I_{\max}}{I_{\min}} = 13 \cdots 156$
- Typical display contrast
  - Cathode ray tube 100:1
  - Print on paper 10:1
- Suggests uniform quantization in the $\log(I)$ domain

Gamma characteristic

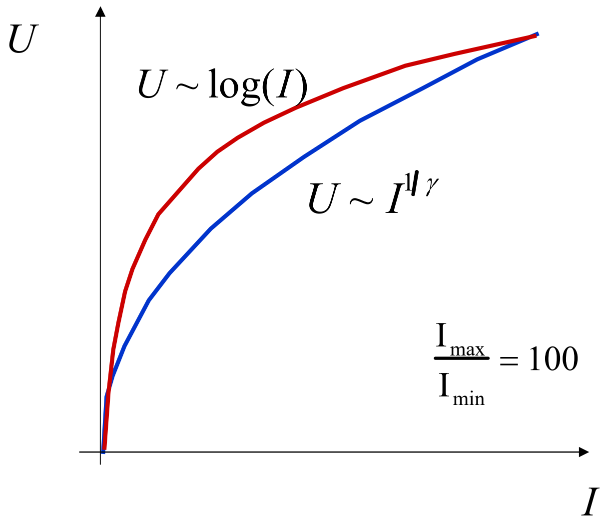
- Cathode ray tubes (CRT) are nonlinear



- Cameras contain γ -predistortion circuit

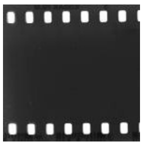
$$U \sim I^{1/\gamma}$$

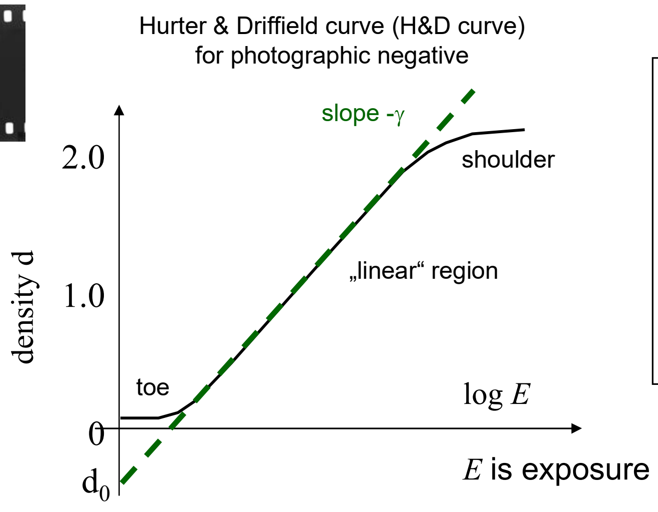
log vs. γ-predistortion



- Similar enough for most practical applications

Photographic film





Luminance

$$I = I_0 \cdot 10^{-d}$$
$$= I_0 \cdot 10^{-(-\gamma \log E + d_0)}$$
$$= I_0 \cdot 10^{-d_0} \cdot E^{\gamma}$$

- γ measures film contrast
  - General purpose films: γ = -0.7 . . . -1.0
  - High-contrast films: γ = -1.5 . . . -10
- Lower speed films tend to have higher absolute γ